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Relationships Between Prior Experiences, Current Teaching Contexts, and Novice Teachers' Use of Concrete Representation for Mathematics Instruction

Elizabeth Lee Johnson
University of South Carolina - Columbia

RELATIONSHIPS BETWEEN PRIOR EXPERIENCES, CURRENT TEACHING CONTEXTS, AND NOVICE TEACHERS' USE OF CONCRETE REPRESENTATION FOR MATHEMATICS INSTRUCTION

by

Elizabeth Lee Johnson

Bachelor of Science
Winthrop College, 1984

Master of Education
Winthrop University, 1992

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Accepted by:

Thomas Hodges, Major Professor

Amy Donnelly, Committee Member

Jan Yow, Committee Member

Debi Mink, Committee Member

Lacy Ford, Vice Provost and Dean of Graduate Studies

DEDICATION

To my daughter, *Steigen Meredith Johnson*, who brings light to my life every day I live it.

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The writing of this dissertation would not have been possible if not for the help of so many wonderful people along the way. I would like to give special thanks to the following people.

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ABSTRACT

This case study documents the perceived influences on three novice elementary teachers' use of concrete representations for teaching mathematics. In order to develop mathematical proficiency, students need access to a variety of representations (i.e. pictures, words, symbols, concrete materials, and real world contexts) to make sense of mathematical ideas. Three sources of data were collected: video-recorded lessons, interviews, and a focus group. Analyses indicated that although concrete representations were accessible to all three teachers, they were least used among the available representations. Verbal expression was most prominent, followed closely by abstract written symbols. Technology, which was not one of the mathematical representations reported, appeared regularly during observations and interviews. Although participating teachers shared similar pre-service experiences in relation to their coursework and internship, there were substantive differences between them in relation to how they viewed adopted standards documents, interacted with colleagues, perceived their students, and perceived district involvement. All three participants expressed concerns related to instructional time, and that district-led professional development was not helpful in supporting the use of concrete representations. The implications in this study are that 1) technology is becoming more visible in classrooms as one form of mathematical representation, and 2) issues relating to CCSSM such as textbook adoption and professional development are considerations impacting instructional practices.

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Chapter I

INTRODUCTION AND OVERVIEW

Introduction

The mathematics that students need to know today is different from the knowledge needed by their parents. In our global society and a world of ever changing technology, when our current students become adults they will face new demands for mathematical proficiency (Casner-Lotto & Barrington, 2006). The mathematics curriculum from Pre-K through middle school has many components, but the heart of mathematics in those years are the concepts relating to numbers and operations. Mathematical proficiency, however, involves many dimensions. In order to have true mathematical proficiency, learners must have conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001).

The Standards for Mathematical Practice describe (CCSSM, 2010) the varieties of expertise that mathematics teachers should seek to develop in learners. These practices are based in part on the Process Standards included the National Council of Teachers of Mathematics (2000) *Principles and Standards for School Mathematics* – reasoning and proof, communications, problem solving, making connections, and using representations. Mathematically proficient students have the ability to make sense of relationships in problem solving. Quantitative reasoning requires the learner to create a meaningful representation of a given problem, consider its parts, think about the meaning of quantities, and use different objects and properties of operations (CCSSM, 2010). Manipulatives are effective learning tools in helping to develop quantitative reasoning. The use of multiple representations and the ability to translate fluently among those different forms facilitates student learning and helps deepen mathematical understanding (Lesh, 1987).

Teachers work with diverse populations using a variety of instructional strategies to meet the needs of their students and insure mathematical proficiency. But this is a task that does not come without specific content and pedagogical knowledge. Teacher education and induction years experiences play an important role in supporting teacher growth and arming teachers with the strategies needed to be effective mathematics instructors.

This study involved analysis of three beginning teachers' practices relating to the use of representations in their mathematics instruction. All three teachers had participated in similar coursework, a field placement, and full-time internship semester during their pre-service education. During the study, these three beginning teachers participated in two interviews (the first during fall semester and the second during spring semester), video-recorded ten mathematics lessons (five during fall semester and five during spring semester), and participated in a focus group discussion at the end of the school year. The discussions were designed to investigate beginning teachers' beliefs about their classroom practices relating to traditional practices versus reform-based options for teaching mathematics. Discussion topics included their pre-service and current teaching experiences in mathematics, their perception of their use of math manipulatives, and their perceptions of their current teaching contexts. The objective was to investigate what influences beginning teachers' implementation of mathematics education reform practices in their classroom as novice teachers.

This study will examine how beginning teachers make use of concrete representations for teaching mathematics and in what ways prior experiences and current teaching contexts impact beginning teachers' use of them. Focusing on the use of math manipulatives for teaching mathematics, this study involves a design that will encompass a collection of video-recorded observations, interviews, and a focus group. The data collected will be used to provide in-depth information about how new teachers use manipulatives in the classroom and how these practices are influenced by their pre-service experiences and current teaching contexts.

Teacher Education

Teacher education for the early childhood and elementary major refers to the guidelines and processes designed to provide prospective K-6 teachers with the knowledge, attitudes, skills, and behaviors required to perform teaching tasks effectively in the classroom. Teacher education is a comprehensive program which combines necessary coursework with a variety of clinical experiences to prepare pre-service teachers with the content knowledge and pedagogical knowledge to be effective classroom teachers. Pre-service teachers take mathematical content courses that include discrete mathematics, data analysis, geometry, and algebraic concepts. In addition to content areas, pre-service teachers take additional coursework relating to effective pedagogical practices through courses such as observation and analysis, technological resources for teachers, child growth and development, educational psychology, working with exceptional and diverse learners, etc. Once pre-service teachers have taken core coursework to build a foundation for solid pedagogical practices, they participate in a series of clinical experiences (often referred to as practicums, field experiences, or internships) under the supervisions of an experienced mentor and/or supervisor. The purpose for these clinical experiences is to allow pre-service teachers an opportunity to gain experience in interacting with learners by applying content knowledge and pedagogical knowledge in an actual teaching environment.

Researchers have recently examined instruction more closely by investigating the choice and use of various academic tasks. Mathematics has many types and levels of representation which build upon one another as mathematical ideas become more abstract (Kaput, 1987; Lesh 1987). Physical representations serve as tools to mathematical thought and communication. They help illuminate ideas in ways that support reasoning and build comprehension. Mathematics requires representations. It is because of the abstract nature of mathematics that people have access to mathematical ideas only through representations (Duvall, 1999.)

Over the past decade, public dissatisfaction with our educational system has included discontent with teacher education (Darling-Hammond, 2000; Stigler & Hiebert, 1999). Ideally, teacher education should be thought of as a seamless process that moves between pre-service coursework and clinical opportunities that provide for practices of knowledge and learned pedagogical practices. Historically, however, our system for educating teachers has been a linear one that begins with university general education coursework, moves to pre-service coursework, ends with a clinical experiences, and releases teachers independently into the classroom. At a time when mathematics education concerns have reached a critical point publicly and politically, it is apparent that our country has a responsibility to offer guidance in answering questions about how to improve mathematics education for all students. This study seeks to consider teacher preparation experiences, yet to look beyond those experiences and consider current teaching contexts such as sources of support, materials, professional development, etc. as possible influences for current teaching practices.

The Importance of Representation in Teaching Mathematics

Mathematical thinking can be "represented" in many ways. It can be represented through drawings and pictures, written or oral words, through manipulatives and, all of these, alongside the abstract (numbers) (Lesh, 2003; Goldin & Shteingold, 2001; Kamii, Kirkland, & Lewis, 2001; Sowell, 1989). Since the early 1900s, representation using manipulatives has come to be considered essential in teaching mathematics at the early childhood/elementary school level, prompting the National Council for Teachers of Mathematics (NCTM) in the last several decades to recommend the use of manipulatives in teaching mathematical concepts (NCTM 1991, 1999, 2000, 2006). Research from both theory and classroom studies has shown that the use of manipulatives for instruction in mathematics can positively affect student learning (Cass et al., 2003; Kelly, 2006; Munger, 2007; Olkun & Toluk, 2004; Ruzic and O'Connell, 2001; Sowell, 1989). The *Principles and Standards for School Mathematics* (NCTM, 2000) highlight the role of representation in mathematics asserting that students should create representations and use them to form and communicate mathematical ideas. Learners should be able to choose, apply, and translate among mathematical representations to solve problems. Greens and Findell (1999) asserted that the use of pictorial, graphic and symbolic representations improved students' ability to interpret algebraic equations. Carbonneau, et al (2012) found that the use of manipulatives improved retention of mathematical ideas. Incorporating math manipulatives into mathematics lessons in meaningful ways can help students grasp mathematical concepts, making graphic pictorial representations more meaningful and making teaching most effective.

To gain better insight into this topic, I used constructivism as a theoretical foundation and Lesh's model of translations of representations (2003) to guide this study. Ideally, students move within and among five forms of mathematical representation in order to construct meaning of mathematical concepts (Cramer, 2003). Representational fluency is the ability to use several different representations and translate among these models with relative ease. This ability is foundational in students' mathematical proficiency (Fennell and Rowan 2001; Goldin and Shteingold 2001; Lamon 2001).

How a teacher facilitates the use of those representations is dependent on many factors such as their pre-service experiences in mathematics, their content knowledge in mathematics (Ball et al., 2005; Burton, 2006), their beliefs about how mathematics should be taught (Ernest, 1989; Hart, 2002; Wilkins; 2008), and their current teaching contexts. The beginning teacher enters the classroom with a set of notions that have been developed about her own ability in mathematics. She has been prepared for teaching during a pre-service university experience by a professor during coursework and a mentor during clinical experiences. She works within a teaching context that will vary among schools and districts. All of these factors will have an influence on the beginning teacher's practices in mathematics instruction (Cohen & Ball, 1990; Hart, 2004; Hill et al., 2005; Wilkins, 2008). The role of the instructor is to cultivate an environment that is conducive to active participation and construction of knowledge. In order to discover how to prepare future classroom teachers to teach mathematics effectively, this qualitative study will explore the teaching practices of new teachers and the factors that influence those practices.

Statement of the Problem

Children in the United States are not developing acceptable levels of proficiency in mathematics (National Center for Educational Statistics, 1999; US Dept. of Education, 2002; The Business Roundtable, 2005; TIMSS, 2007; National Mathematics Advisory Panel, 2008; The Nation's Report Card, 2011). Concern about achievement in mathematics of US students has grown, and research has shown that improvement in the teaching and learning of mathematics is needed (Ball, Hill, & Bass, 2005; Committee on Science and Mathematics Teacher Preparation, 2001; Fleishman, et al, 2010; Gonzales, 2007; National Center for Educational Studies, 1999). Results from the Trends in International Math and Science Study (TIMSS) Report in 2007, served to reinforce that concern regarding how mathematics is taught in the US. The scores from the 2009 Program for International Student Assessment (PISA) show 15-year-old students in the US scoring below average in mathematics. Out of 34 countries, the US ranked 25th in math. Only 27% of US students scored at or above proficiency level 4, which is the level at which students can complete higher order tasks such as solving problems that involve spatial reasoning in unfamiliar context and carrying out sequential processes (The Nation's Report Card, 2011). A mere forty-two percent of students in the US who took the ACT (American College Testing) in 2006 scored met or exceeds the College Readiness Benchmark. These results have brought about a need for reform in how students are taught and how teachers are prepared to teach them. There needs to be an emphasis on representational fluency in our classrooms in order to help our students gain mathematical understanding on a much deeper level.

Purpose Statement

The National Council of Teachers of Mathematics Standards (2000) and the Common Core State Standards of Mathematics (2010) stipulate that students need opportunities to solve problems and have multiple occasions to communicate mathematical ideas with others, and that teachers should focus on student understanding rather than on "right" answers. This stance toward teaching mathematics represents fundamental changes in teaching practices, shifting away from the more exclusively traditional (to be defined at the end of this chapter) practices (Stipek, et al, 2001). The standards are reiterated and revised year after years, yet the reforms do not seem to be taking hold in our mathematics classrooms.

The National Mathematics Advisory Panel (2008) recommends that more research is needed in the area of teaching and learning mathematics. Specifically, research should look at effective instructional practices, mechanisms of learning, ways to enhance teachers' effectiveness (including teacher education that is directly tied to student achievement), and test features that improve assessment of mathematical knowledge (2008). In light of this recommendation, this study seeks to investigate what influences teacher practices in mathematics instruction.

The purpose of this study is twofold. The first is to investigate whether three new teachers use math manipulatives to help young mathematicians conceptualize sophisticated mathematical concepts. The second is to identify the relationship between concrete representation use and experiences which may influence representational choices. Should a relationship be suggested between how new teachers were taught and how they use manipulatives in the classroom, further study on a larger scale would be warranted to determine whether these same patterns occur in a larger sample and *how* various manipulatives are being used. The findings from this small-scale investigation will help inform instruction and curriculum surrounding mathematics methods at one university and determine the design of a larger-scale investigation to identify a link between new teachers' instructional practices in math and their pre-service preparation.

Research Questions

To address the aforementioned purpose, this study ask two questions:

1) How do beginning teachers make use of concrete representations for teaching mathematics,
2) In what ways do prior experiences and current teaching contexts impact beginning teachers' use of concrete representations?

Significance of the Study

Many studies have investigated knowledge and experiences new teachers bring to the classroom from their pre-service experiences and have addressed mathematical content knowledge for elementary mathematics (Ball, 1990; Ball & Hill, 2004; Hill et al, 2007). Although these studies have shown that content knowledge has a significant impact on new teachers' ability to effectively teach mathematics, content knowledge alone is not enough (Ball & Hill, 2004). New teachers' self-efficacy and comfort level with mathematics instruction also plays an important role (Burton, 2006; Burton & Geddings, 2008; Hart, 2004; Philipp, 2007; Swars, 2007; Wilkins, 2008). The time that pre-service teachers transition from the university class as a student to the elementary classroom as a teacher is important because it is this time of practicing previously learned instructional strategies that teachers solidify their pedagogical practices (Ball, et.al. 2001; Ball & Hill, 2004; Ball, et.al. 2005; Hill, et al, 2007).

The Curriculum Focal Points (NCTM, 2006) and the Principles to Action (2014) documents call for concrete representations to be used in teaching a variety of concepts in mathematics. The use of manipulatives is recommended because it is supported by both learning theory and educational research in the classroom (Baker, 2008; Hartshorn & Boren, 2000; Moyer & Jones, 2004; Ruzic and O'Connell, 2001). Manipulatives can be a key to providing effective, engaging lessons in mathematics instruction. Manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning (Heddins. 1986; Ross and Kurtz, 1993). Mathematical learning takes place in many stages, concrete through real-life experiences and manipulatives, graphic through pictures, drawings, and graphs, linguistically through written and spoken word, and abstract through numbers, letters, and symbols (Anstrom, 2000; Bender, 2009; Bley, 2001; Hardy, 2006; Lesh et al., 1987; Maccini& Gagnon, 2001; Souza, 2008). In order for students to move through the stages of learning and arrive at a deep understanding of mathematical concepts, teachers play a critical role in helping students use manipulatives successfully.

To address this study, I used Lesh's Model of Mathematical Representations and Translations (1987) as the basis for my observations. I incorporated part of Borko's Scoop Notebook (2007) to collect video recorded data prior to conducting interviews and a focus group. My study investigates how new teachers' preparation to teach mathematics prior to entering their own classroom and their current teaching contexts influence their current teaching practices. The results of this study will help inform the design of pre-service professional development approaches which should increase inquiry-based teaching of mathematics in the classroom. The results may also be useful in helping new teachers transition into the profession using instructional practices that are familiar, theoretically congruent, and are used consistently within the classroom setting in order to increase students' mathematical understanding and improve student scores in mathematics.

Limitations of the Study

Due to the small and unique sample size, the ability to generalize these results beyond the specific population to other areas and other states may be limited because it was conducted through one university in South Carolina. If however, the findings suggest a relationship in this small sample, reliability would be assured by further research of a similar nature but with a wider scope. Because this is a snapshot of the influence of pre-service and current contexts, this study will assess present status but may not be valid to predict future success.

Finally, it is important to disclose that there is a possibility that the researcher has taught some of the participants, as the researcher has been 1) an instructor of the early childhood math methods course, 2) an instructor of the senior seminar for education majors ranging from early childhood to secondary education, and 3) a supervisor of early childhood and elementary interns during the time frame in which some of the participants may have been enrolled in courses at the university.

Delimitations of the Study

This study is limited in that it consists of three participants who shared similar pre-service teacher preparation experiences. The extent of the geographic region from which data are collected are limited as all three participants teaching locations were within a geographical radius from the university of 45 miles.

In order to assure manageability of the collected data, data collection instruments were limited to two one-on-one interviews per participant, ten days of video-recorded mathematics lessons being taught by each of the participants, and one focus group which included all three participants.

Definition of Terms

Manipulatives. Manipulatives, also referred to as **concrete representations**, are objects designed to allow students to learn a particular mathematical concept by manipulating them (Reys et al, 2007; Krech, 2000; Van de Walle, 2005). The use of manipulatives allows students to learn difficult concepts in developmentally appropriate, hands-on, experiential ways (Reys et al, 2007). Some examples of manipulatives are: base ten blocks (which can be used for computational strategies with whole numbers and decimals); Geoboards (which can be used to explore two dimensional geometric shapes, area and perimeter, angles, etc.); pattern blocks (which are used in creating tessellations and exploring patterns in our world around us); fraction pieces (which can be used for computational strategies with fractions); and attribute blocks (which are effective in categorizing and organizing by characteristics). With research supporting the use of manipulatives to increase student learning and problem solving (Raphael & Wahlstrom, 1989; Sowell, 1989), mathematics manipulatives are common in K-8 classrooms.

Pictorial representation. Pictorial representation, also referred to as pictures, refers to anything hand-sketched or computer generated that represents concrete objects. It could be a photograph, a hand-drawn picture, tallies, graph, or chart. These may include any two-dimensional representations (Ainsworth, 1999; Tabachneck-Schijf & Simon, 1998).

Real-life representation. Real-life representation refers to events and objects happening in the real world that allow students to make mathematical connections. Examples may include using money in a grocery store, measuring ingredients when cooking a recipe, or measuring wooden beams when building a garage, etc. (Lesh, 1987).

Symbolic representation. Symbolic representation, also referred to within this study as symbols, refers to the actual letters, digits, and/or symbols used to represent numbers, formulas, or any other numerical, algebraic, or geometric concepts (Ainsworth, 1999; Tabachneck-Schijf & Simon, 1998).

Constructivist classroom. A constructivist classroom, often used interchangeably with experiential or inquiry-based classroom, is one which allows the learner to construct learning through a variety of different experiences (Montigue, 2003; Norton & D'Ambrosio, 2008). Constructivism is a type of learning theory that explains learning as an active attempt to construct meaning from what is around us (Brooks & Brooks, 1993). Constructivists believe that learning is an active process and self-directed, as opposed to passive and teacher-directed (Brooks & Brooks, 1993; Bruner, 1991; Hsueh, 2005; Noddings, 1990; von Glasersfeld, 1995).

Traditional classroom. A traditional classroom, as used in this study, is one where the learner takes a more passive role and in which the teacher lectures or "transmits" information to students (Santrock, 2007). It often is exemplified with minimal interaction between students and the teacher, or students with other students, and tends to use secondary sources for information (such as textbooks), and uses more pencil/paper instruction and assessment (Jackson, 1986).

Field experience. Field experience is the placement of a teacher candidate in the classroom with an assigned mentor teacher for 2.5 days a week during a 14-week academic semester. During this time, the teacher candidate becomes familiar with the classroom demographics, roles and routines, and instructional and assessment practices. Teacher candidates take their math methods class during this semester, then practice planning and implementing in the classroom the strategies that they are learning (Winthrop University, 2011).

Internship. An internship is the placement of a teacher candidate in the classroom with the same assigned mentor teacher for five days a week during a 14-week academic semester. During this time, the teacher candidate teaches on a regular basis at the beginning of the semester with the goal of taking full time responsibilities, with the assistance and oversight of the mentor, toward the end of semester. Teacher candidates have completed their math methods class by this semester, so the ongoing weekly reinforcement of university-taught coursework is no longer in place. Teacher candidates plan and implement instruction in all subject areas with mentor assistance as needed (Winthrop University, 2011).

Beginning Teachers. For the purpose of this study, beginning teachers are defined as those teachers who have been teaching in their own classroom for three years or less.

Mentor. The *mentor* is the individual employed by the school district to work in partnership with the university to assist the teacher candidate during the field and internship experiences. The role of the mentor is to work with the teacher candidate in the planning, implementation, and assessment of lessons, conference with teacher candidate and supervisor as needed, complete a minimum of two formal observations during the field experience and three formal observations during the internship, provide regular feedback to the intern, formally meet with the intern and supervisor for a midpoint evaluation and final evaluation in each of the above experiences, and provide formative assessment throughout each semester (Winthrop University, 2011).

Organization of the Study

This study is organized into five chapters. Chapter I provides an introduction to the study, including a statement of the problem, purpose statement, and research questions. The significance of the study, limitations and delimitations of the study are discussed, and definitions of the terms are briefly defined. Chapter II includes a review of the literature related to constructivism and mathematics representations. A historical perspective of reforms in mathematics included efforts put for by the National Council of Teachers of Mathematics, the Common Core State Standards, and independent educators supporting reforms in teacher development. Chapter III describes the study's research methods, including the research design and a description of the participants and the context of the study. The data collection process is discussed, and a brief description of the data analysis is introduced. Chapter IV outlines the results of the study by reporting on each of the two research questions, *"How do beginning teachers make use of concrete representations for teaching mathematics"* and *"In what ways do prior experiences and current teaching contexts impact beginning teachers' use of concrete representations?"* This chapter includes the statistical data to provide evidence that supports the inquiry into the first question and sample dialogue to support the thematic analysis for the second question.

Finally, Chapter V includes a brief overview of the study, detailed data analysis and results. Relationships of the findings between the two questions were explored. Implications of the study were examined, and how study results relate to teacher education and postsecondary involvement was explored. I concluded the chapter with recommendations for future research.

Chapter II

REVIEW OF THE LITERATURE

Introduction

A mathematics teacher's instructional practices are influenced by many different factors. To gain better insight into this topic, I used constructivism as a theoretical foundation and Lesh's model of translations of representations (1987) to guide this study. This chapter contains an overview of the literature related the study. It begins with an examination of instruction from a constructivist perspective. It then investigates models of mathematics representation and how the use of those mathematical representations affect student learning. The next section takes a historical look at mathematics reform over the past two decades, including guiding policy documents which inform the standards and curriculum adopted in the elementary schools. This impacts the need for further teacher development. The final section very briefly discusses teacher beliefs and practices as they might relate to study participants. This literature situates this study within a global context that illustrates the need to support teachers' implementation of complex mathematical thinking through the use of high demand tasks that afford teachers and students opportunities to represent mathematics in a variety of ways.

Examination of Instruction from a Constructivist Perspective

Traditionally, many schools have adopted the practice of transmission of ideas, an instructional model in which the teacher lectures or "transmits" information to students (Santrock, 2007). Mathematics reforms call for classrooms that move away from the traditional "transmission of ideas" type of classroom to one that utilizes inquiry, discovery, representations, and discourse (Goldin & Shteingold, 2001; Loveless, 2001). Broadly, reforms in mathematics education follow constructivist views of teaching and learning, which states learning as an active attempt to construct meaning from what is around us, and is self-directed as opposed to passive and teacher-directed (Montigue, 2003; Brooks & Brooks, 1993; Bruner, 1991; Hsueh, 2005; Noddings, 1995; von Glasersfeld, 1995). Under this view, the teacher's responsibility is to understand the connection between how individuals learn, what instructional needs are present, then make decisions on the use of various strategies to best accommodate learning. In order for education to be effective, content and experiences must be presented in such a way that learners can connect new information to prior experiences (Bruner, 1991), allowing them to construct their own learning and deepening the connection with new knowledge (Dewey, 1902). Representations help develop such connections through presenting real-life situations, concrete representations, pictures, and verbal and written words to support mathematical learning (Lesh, 2003).

Mental functions, such as representing thoughts in pictures and writing, develop through social interactions with "more knowledgeable others" and experiences in a child's life (Vygostsky, 1978). Through these interactions, children come to learn their culture, such as speech patterns, written language, and other symbolic representations through which children gain meaning. This learning affects the construction of knowledge where it can be internalized (Santrock, 2007). Students play an active role in their learning, and social interaction plays a fundamental role in cognitive development (Vygotsky, 1978).

Many researchers have advocated active involvement of children in the learning process (Boggan, et al., 2010; Hartshorn & Boren, 2000; Saettler, 1990; Ward, 1971).

Constructivist researchers have contributed to the popularity of the use of mathematical representations. Dienes' (1969) work showed that the use of various representations of a concept were needed to support students' understanding. Piaget (1952) suggested that children do not have the mental maturity to truly grasp abstract mathematical concepts that were presented merely in words or symbols alone, and that children need many and a variety of experiences with concrete materials and graphics in order for learning to occur. Jerome Bruner (1966) described three stages of representation: a) the enactive, or action-based, stage often referred to as the concrete stage which involves tangible hands-on method of learning, b) the iconic, or image-based, stage which is sometimes called the pictorial stage and involves images or visuals to represent concrete situation enacted, and c) the symbolic, or language-based, stage which takes the images from the first stage and represents them using words and symbols. These stages of representation offer an opportunity for learners to connect real-life objects to more abstract concepts in order to make meaning and facilitate learning. Each stage acts as scaffolding for the next stage which builds off of the information acquired in the previous stage (Culatta, 2012; Smith, 2002).

Educational research indicates that learning occurs as students actively participate and assimilate information and experiences and construct their own meaning (Ball, 2003; Case and Bereiter 1984; Davis 1984; Heibert 1986; Hill et al, 2007; Hill et al, 2005; Lampert 1985; Schoenfeld 1987). Current research in mathematics views learners as active participants who construct their own knowledge and understanding of concepts by reorganizing current ways of knowing and deriving meaning from their own experiences (Moyer, 2001; Simon, 1995; von Glaserfeld, 1995). The impact of this research, which connects students' actions with physical objects to their mathematical learning, has an influence on the use of manipulatives in the classroom.

Mathematics Representation

Conceptual understanding is essential to mathematics proficiency (Battista, 1999; Burns, 2005; Hiebert & Wearne, 1992). Abstract mathematical concepts often create challenges for students in constructing mathematical understanding (Devlin, 2000; Kamii et al., 2001), therefore hands-on manipulatives and graphic pictorial representations of mathematical concepts are helpful. Hands-on experiences allow students to understand how numerical symbols and abstract equations operate at a concrete level (Devlin, 2002; Maccini & Gagnon, 2001). Students can benefit significantly from instruction that includes multiple models that approach a concept at different cognitive levels (Lesh & Doerr, 2003; Lesh et al., 2003; Lesh & Fennewald, 2010).

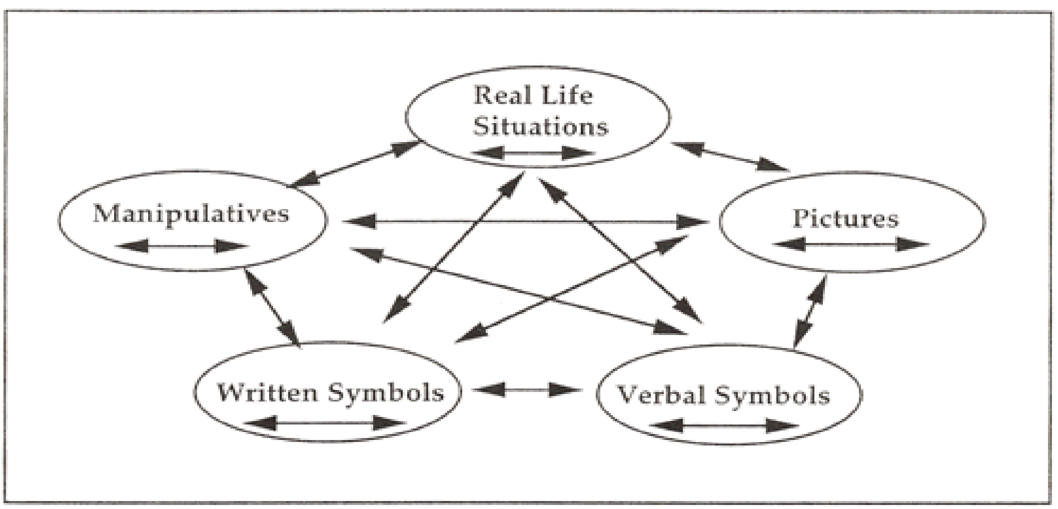

Figure 2.1 Lesh Translation Model

Building on Bruner's stages of representations, Lesh, Post, and Behr (1987) identified five types of representation that occur in mathematics. Each of the five representations is interactive with the others, and none takes precedence over another. Theoretically students move fluidly among and between the representations. The first are experiences in which knowledge is organized around "real world" events that provide a context for interpreting and solving other kinds of problems. The second are concrete models, or manipulatives (such as base ten blocks, Cuisenaire rods, arithmetic counters, fraction pieces, Geoboards, etc.), which can be used for mathematical problem solving and everyday situations. The third type of representations are pictures or diagrams that, like manipulatives, can be internalized as in order to construct mathematical meaning. The fourth is spoken language, which may include specialized sub languages related to domains like problem solving, logic, etc., and the fifth is written symbols which, like spoken language, can involve specialized sentences and phrases appropriate to sub languages. This model demonstrated students moving among the representations as opposed to a sequential usage, therefore transformations and translations tend to be interdependent.

One major conclusion from Lesh et al. (1987) is that students have seriously deficient understandings in the context of word problems and pencil and paper computations. Many have equal difficulty understanding about the models and language needed to represent and manipulate these ideas. (Behr, Lesh, Post, & Wachsmuth, 1985; Post, 1986). To identify instructional opportunities teachers can generate a variety of questions by presenting an idea in one representational mode, and having the student illustrate, describe, or represent the same idea in a different mode. If questioning indicates difficulties in one mode or process, other processes in the diagram can be used to strengthen or bypass it (i.e. a child who has difficulty translating from real situations to written symbols might begin by translating from real situations to spoken words and then translate from spoken words to written symbols; or it may be to use manipulative models and translate meaning into representation and subsequently construct abstract knowledge.)

Lesh contends that students who are proficient in mathematics are able to translate from one mode of representation to another as a means of demonstrating understanding of a problem (for example "build this using concrete objects" or "say it in your own words" or "can you think if as similar problem?") His research suggests that the act of representation tends to be "*plural*, *unstable*, and *evolving"*; and these three attributes play important roles to make it possible for concepts and representations to evolve during the course of problem-solving sessions" (Lesh et al., 1987, p.37). He argues that not only might problems occur naturally in the form of many modes but solution paths will also often weave back and forth between several representational systems (Lesh, Landau, & Hamilton, 1983). The Lesh Translation Model depicts the importance of learners being able to represent mathematical concepts in multiple ways, including with real-life situations, manipulatives, pictures, and verbal and written symbols (Lesh et al., 2003).

In the book *Brain Matters: Translating Research Into Classroom Practice*, Wolfe (2001) contended that concrete experiences provide meaning to the learner, since the representational or symbolic experiences may have little meaning without the concrete experiences on which to build. It is difficult for teachers to help students make connections because mathematics is so abstract (Hartshorn & Boren, 2000). One way of making those abstract concepts more concrete is the use of math manipulatives, objects designed to concretely represent mathematical ideas and concepts (Moyer, 2001), for mathematics instruction. Manipulatives are often described as physical objects that are used as teaching tools to engage students in hands-on exploration of mathematical concepts (Boggan, et al., 2010).

Research indicates that the proper use of manipulatives results in marked success in achievement and that manipulatives are particularly helpful in assisting students in understanding mathematical concepts (Hartshorn & Boren, 2000; Raphael & Wahlstrom, 1989; Sowell, 1989). Ball (1988) found that fourth grade students that used both physical manipulatives and virtual manipulatives scored significantly higher when tested on conceptual understanding of fractions than students who had not used manipulatives. Bruner (1987) concluded that children demonstrate their understanding in three stages of representation and suggested the use of physical objects prior to moving to the use of symbols. Based on earlier studies of Vygotsky (1978), Cobb (1995) discusses mathematical tools and the connection between mathematics manipulatives and socio-cultural perspectives. Research by Skemp (1987) supports the belief that students' early interaction and experience with physical objects provided a foundation and a basis for later, more abstract, learning.

Chester, et al (1991) studied two third-grade classes where manipulatives were used with the experimental group and a more traditional approach was used with the control group to teach a geometry unit. She found that the experimental group scored significantly higher than the control group on the posttest. Cobb (1995) did a case study of four pairs of second graders who were beginning to learn about place value and noted that the building of our notational system, and the manipulatives we use to convey it, help to organize our math experience. Cotter's (2000) study observed mathematics instruction in two first grade classrooms, one using base ten blocks and abacuses for instruction and the other using traditional approach of lecture and textbook. She concluded that the group using manipulatives exhibited a better conceptual understanding of place value. Ruzic and O'Connell (2001) found that the long-term use of manipulatives for mathematical instruction has a positive effect on student learning by allowing students to use concrete objects to observe, model, and internalize the more abstract concepts. Sutton and Krueger (2002) found that the use of manipulatives engaged students and increased their interest and enjoyment of mathematics, resulting in higher achievement. Munger (2007) reported that the experimental group using math manipulatives scored significantly higher in mathematical achievement than the control group.

In a survey of classroom teachers conducted by the National Education Association in 2002, 85% of elementary teachers rated the use of math manipulatives "highly effective." Sowell (1989) combined the results of 60 studies to determine the effectiveness of mathematics instruction with manipulatives and found that long-term use of manipulatives was more effective than short-term use because it allowed students to use concrete objects to model, observe, and internalize abstract concepts. Similarly, Ruzic and O'Connell (2001) identified 11 research studies that were published between 1992 and 1999 on the use of manipulatives in the classroom and found that the use of manipulatives for mathematical instruction as compared to traditional instruction typically had a positive effect on student achievement. Cramer, et al. (2002) compared the achievement of students using a commercial curriculum for learning fractions with the achievement of students exposed to a manipulative-based curriculum that put greater emphasis on the use of manipulatives. Students using the manipulative-based curriculum have statistically higher average scores on post- and retention tests.

Baker (2008) found that the benefits of using math manipulatives in elementary and secondary classrooms influenced both students and pre-service teachers. Student benefits included improved student content mastery and increased student engagement. Benefits for pre-service teachers included reduction of anxiety and change in attitude toward mathematics. Ball (1993) concludes that it is through the manipulatives' use as tools that students are able to gain insight into their experiences with them. Boulton-Lewis (1998) contends that for learners to use concrete representations effectively, they must be familiar enough with the materials to use them automatically. It is the mediation by teachers and students in shared discussion and meaningful practices that determine the usefulness of the manipulatives (Moyer, 2001).

The revision of the NCTM Standards, Principles and Standards for School Mathematics (2000) discussed in the previous section, recommends extensive use of manipulatives, particularly in the earlier grades. Teachers need to provide the necessary manipulatives, graphic illustrations, and experiences to allow students to build and explore in order to develop mathematical concepts. Learners must reflect on and communicate their experiences with manipulatives in order to build meaning (Kelly, 2006; Krech, 2000). Even if teachers have learned appropriate strategies for using manipulatives for mathematical instruction, their beliefs about how students learn mathematics may influence how and why teachers use manipulatives the way that they do (Moyer, 2001; Philipp et al. 2007).

Historical Perspective of Reform in Mathematics

National Council of Teachers of Mathematics (NCTM)

In an effort to provide a guide for states' development of standards for mathematics, the National Council of Teachers of Mathematics (NCTM) took steps to develop a set of standards which emphasized problem solving and connectedness among mathematical ideas. The outcome was the NCTM Curriculum and Evaluation Standards (1989), commonly known as NCTM Standards. The National Science Foundation (NSF) was in clear support of these standards, feeling that students should learn through group-based discovery with the assistance of manipulatives and calculators, and supported the NCTM Standards implementation with Statewide Initiative Awards. The NSF supported the development of mathematics curricula aligned to the NCTM Standards (Klein, 2002). Throughout the 1990s, the NSF sponsored the creation of several elementary mathematics programs including "Everyday Math," which is still in use today (Klein, 2002). By the mid-90s, however, due to falling test scores in math, a growing controversy involving parents, educators, mathematicians, education reformers, school boards, government policy-makers, and politicians known as the "Math Wars" was well underway. The 1989 NCTM Standards met with harsh criticism from American Mathematical Society (AMS) and the Mathematical Association of America (MAA) (Loveless, 2001). For a brief period, the fate of the NCTM Standards rested in its ability to redraft a set of standards that would satisfy parents, educators, and mathematicians and improve future test scores, the National Council of Teachers of Mathematics set out to revise the 1989 standards.

The National Council of Teachers of Mathematics Standards (1991) are a set of principles that serve as guidelines to judge what is valuable and appropriate in mathematics instructional practices. These standards give direction to educators interested in improving mathematical education, from the classroom teachers, to administrators, to schools and school districts, to universities, to state departments and professional teaching organizations. They suggest a movement away from traditional pencil/paper calculations and a move to inquiry-based approaches to learning and teaching. Based on the NCTM Standards (1991), there are five major shifts in the environment of mathematics classrooms that are needed to move away from traditional practices and toward mathematics teaching that empowers the students. I created Table 2.1 to summarize these shifts.

Table 2.1 Comparison of Traditional Practice vs Reform-based Practice

A shift away from:	**A shift toward**:
classrooms in which students are simply individuals	math communities where mathematical discourse is prevalent
teachers as sole authority	logical and mathematical evidence for verification
simply memorizing	mathematical reasoning
from simple answer-finding by mechanics	invention and problem-solving
isolated concepts	mathematical connections

The Standards for the Professional Development of Teachers of Mathematics (1991) focused on NCTM's vision for well-prepared mathematics teachers by focusing on what a teacher needs to know about mathematics, mathematics education, and pedagogy to carry out its vision of these five shifts. Professional development for pre-service teachers should include modeling good mathematics instruction, knowing mathematics and school mathematics, understanding students as learners of mathematics, practicing quality mathematical pedagogy, developing as a teacher of mathematics, and understanding the teacher's role in professional development (NCTM, 1999). NCTM calls for mathematics classrooms where concept development, problem-solving, and construction of learner-generated solutions are given more emphasis than memorization of algorithms and procedures to get the right answers.

This is a major shift from memorizing facts and procedures to learning and internalizing mathematical content through experience. The kind of teaching envisioned in these standards is different from the kind of teaching that many of our current teachers have experienced in their own mathematics education (NCTM, 2000). The new expectations present a challenge for pre-service teachers who have come through the educational system with the traditional instructional background. Standards (NCTM, 2000) indicate that teachers should help students work together to make sense of difficult mathematical concepts. Teachers should design and implement instruction to help students solve problems and make connections with mathematical ideas and applications (National Council of Teachers of Mathematics, 2006).

Based on the updated NCTM Standards (2000) there continues to be reformed views about how mathematics should be taught (Ball, Hill, & Bass, 2005; Ma, 2010; NCTM, 2006; National Mathematics Advisory Panel, 2008; Philipp, 2007; Wu, 2008). These views reflect movement away from traditional instructional practices in mathematics and toward a more reform based practice. With that shift comes a whole new learning curve for teachers. Many programs take a constructivist approach to mathematics learning and teaching, attempting to provide quality experiences for students that encourage the alignment of beliefs with current reform recommendations (Montigue, 2003; Swars et al, 2007). National Council of Teachers of Mathematics Standards (NCTM, 2000) specify those recommendations, provide a guideline for what students should be learning at specific grade levels, and provide a roadmap for the teachers who are providing instruction. The goal of the standards is to provide guidance to those teaching mathematics in an effort to support reform of mathematics instruction in the schools.

Common Core State Standards in Mathematics (CCSSM)

Similar in one way to the NCTM standards, the Common Core Standards establish clear, concise guidelines for what students should be able to do in mathematics from kindergarten through grade 12 and focuses on developing critical-thinking, problem-solving, and analytical skills. For more than a decade research studies of mathematics education have concluded that mathematics in the U.S. must become more intelligible in order to improve student achievement in mathematics (CCSS, 2010). The CCSS, a state-led effort coordinated by the National Governors Association and the Council of Chief State School Officers, released final versions of the voluntary common standards for English/language arts and mathematics in June 2010. The Common Core State Standards in Mathematics (CCSSM) are designed to provide clarity and specificity instead of broad general statements. The CCSSM follows the design envisioned by William Schmidt and Richard Houang (2002) of a coherent curriculum that stresses conceptual understanding of key ideas while continually returning to foundational organizing principles such as place value and basic laws of arithmetic (CCSSM, 2010). The development of the standards began with research-based learning progressions detailing what we as educators know about how students' mathematical understanding develops over time (CCSSM, 2010). These standards define what students should know and understand in their study of mathematics in both conceptual understanding and procedural skill.

Like the reaction of the initial roll-out of 1989 NCTM Standard, Common Core was also originally met with favor with 46 states and the District of Columbia adopting the standards. However concerns about finding instructional materials aligned to the new standards, possible changes in instructional practices, faculty training, and assessment, several states have reconsidered CCSS adoption. In order to address some of the concerns relating to Common Core, the CCSS Initiative released a fact sheet to clarify some of the questions and misconceptions among parents and educators (CCSS, 2010). The following table addresses the facts and misconceptions about the Common Core State Standards as a whole and specific to mathematics:

Table 2.2 Common Core Misconception vs Fact

Misconception	FACT
Adopting CCSS means bringing all standards down	CCSS is designed to build upon the most advanced requirements
CCSS are not internationally benchmarked	Standards from top-performing countries played a significant role in the development of CCSS
CCSS only include skills and not important content knowledge	CCSS recognize that both content and skills are important
CCSSM do not require students for Algebra I in Grade 8 as other state standards do	CCSS accommodate and prepare students for Algebra 1 in 8th grade by including the prerequisites for this course in grades K-7
In CCSSM Key math topics are missing or appear in the wrong grade	The mathematical progressions presented in the Common Core State Standards are coherent and based on evidence.
CCSS tell teachers how to teach.	CCSS trusts that teachers know best about what works in the classroom, so schools and teachers will decide how best to help students reach the standards.
Teachers will be left to implement the standards without any support or guidance.	Decisions on how to implement the standards are made at the state and local levels.
CCSS will be implemented through No Child Left Behind (NCLB), signifying that the federal government will be leading them.	CCSS is a state-led effort that is not part of No Child Left Behind or any other federal initiative. The federal government played no role in the development of the Common Core.
CCSS amount to a national curriculum for our schools.	The CCSS is *not* a curriculum. It is a clear set of shared goals and expectations for what knowledge and skills will help our students succeed.

The progressions of learning are research-based and coherent (Cogan et al., 2013). Unlike the NCTM standards, CCSS does not address *how* to teach but *what* to teach. It is not a curriculum, but a uniform set of benchmarks that leaves instructional approach to schools and teachers. It is a state-led initiative that is not in partnership with initiatives of the federal government (CCSS, 2010).

There are three primary shifts in the Common Core State Standards for Mathematics that are different from previous standards and that are necessary for effective implementation. The first is a greater concentration on fewer topics. Rather than covering such a wide range of topics that can only be covered at a surface level due to time constraints, CCSSM encourages teachers to narrow and deepen their focus on topics in the classroom. CCSSM gives a few specific areas for each grade range (i.e. concepts, skills, and problem solving in addition and subtraction for grades K-2, multiplication and division for grades 3-5, ratios and early algebraic expressions for grade 6, etc.). The second is linking topics and thinking across the different grade levels. The topics are connected in a progression from one grade level to the next. Coherence is built into the standards by reinforcing a major topic in a grade level by using supporting, complimentary topics. The third shift is in the rigor of the standards. The idea that is emphasized is having teachers address conceptual understanding of key concepts, procedural skills and fluency for accuracy, and correct application all with equal intensity (CCSSM, 2010).

Teacher Education

Because standards for learning are higher than they have ever been before, the importance of producing quality teachers is increasingly important. Pre-service teachers need to develop the skills to pose real-life problems for students to solve and allow them opportunities to pose additional problems (Steele, 2001). Teachers of elementary mathematics have the challenge of putting to use their content knowledge of elementary mathematics as well as their knowledge of pedagogical practices to create an experiential classroom that helps students construct and internalize knowledge of mathematical concepts (Ball, 1988; Ball, 2003; Ball & Bass, 2003; Cochran-Smith, 2001; Scott, 2004). The key to improving the mathematical education of our students is improving the mathematical education of our teachers through university coursework, quality field experiences, and continued practice using sound pedagogical strategies (Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). The weakness of traditional teacher education programs is that many are collections of unrelated courses (Cochran-Smith, 2001). In recent years we have learned a great deal about how to develop more effective teacher education programs (Cochran-Smith, 2001). In spite of criticisms of teacher education, evidence indicates that teachers who have had more time and preparation for teaching are more confident and successful with students than those who have had little or none (Cochran-Smith, 2003).

Research indicates that reforms of teacher education which include extended clinical preparation in conjunction with coursework on learning and teaching creates teachers who are more effective (Darling-Hammond, 2000). Strides toward teacher education reform were made in the late 1980s and early 1990s through programs such as the Holmes Group Report, the Carnegie Forum on Education, the Economy Task Force on Teaching as a Profession when they outlines an agenda for professionalizing teaching, and the National Network for Educational Renewal was created. Many important reforms since have taken place because of these initiatives which strengthened the subject matter and pedagogical preparation of pre-service teachers. The introduction of Professional Development Schools (PDS) partnership is one such example that has changed the nature of training for teachers and created authentic settings for teacher learning (Darling-Hammond, 2000). There are three critical elements to effective teacher education programs: 1) integration among courses and between the coursework and the clinical experiences in the schools, 2) diligently supervised clinical experiences that are closely related to the coursework and that use pedagogies that link theory and practice, and 3) close relationships with schools that serve a diverse population and model quality teaching practices (Darling-Hammond, 2000).

In addition to developing programs that have more integration between coursework and clinical experiences, Cochran-Smith (2003) makes the argument that the education of teacher educators is enriched when inquiry is the stance taken in relation to teaching, schooling, and teacher education. Despite the many expectations that teachers are trying to meet, there has been little attention on the development and adoption of a curriculum for educating teacher educators (Cochran-Smith, 2003). Cochran-Smith (2003) articulates that educating teacher educators should be an ongoing process that depends on inquiry as a stance in teacher education. She adds that many of the professionals who work most closely with teachers during their clinical experiences are not higher education faculty members, but are often part-time, adjunct, temporary, or clinical faculty. She suggests that the process of educating teacher educators should be conceptualized and extended across the professional lifespan and not one that ends when the coursework is completed.

Teacher Development

Central to all of the standards meeting the call for reform is the ability to explore, reason, problem-solve, communicate, and connect ideas. Developing these processes requires teachers who are proficient in engaging students' interests, providing opportunities to deepen understanding, providing for classroom discourse to promote understanding, and helping students seek connections (NCTM, 2000). Guidelines for standards call for actively experiencing good teaching, knowing the content, knowing the students as learners, addressing issues and topics significant in education, knowing pedagogy, engaging in continual development, reflecting, collaborating, and developing the teacher's role in professional development (CCSS, 2010; NCTM, 2000; NCSS, 1994; NRC, 1996).

Teachers must obtain a strong conceptual knowledge of the mathematics that they will be teaching (Ball, 1988; Ball et al., 2001; Ball et al.; 2005; Burton, 2006; Hill et al., 2005; Hill et al., 2008) and must also be prepared to differentiate instruction to accommodate different learning styles and different developmental levels (U.S. Department of Education, 2002). Teacher Education Programs must prepare teachers for these increased demands by providing pre-service teachers with experiences that nudge their mathematical pedagogical beliefs toward alignment with a more reform-based perspective and increase their efficacy for teaching mathematics (Brown & Borko, 1992). Because teacher beliefs are often informed by past experiences (Bandura, 1977), and attitudes toward mathematics are generally focused on formulas and correct answers (McGowen & Davis, 2001; Ma, 1999; Wu, 2008), teacher education programs have an additional challenge in that they must reverse the traditional notion of mathematics instruction and embrace a reform-based perspective. But influencing teacher's beliefs and practices does not stop in the university classroom (Ernest, 1989; Hart, 2004; Holt-Reynolds, 2000).

Many university students, including pre-service teachers, see their job as memorizing the right answers and repeating them back on request (Holt-Reynolds, 2000b; Muis, 2004). It can take time for pre-service teachers to align their pedagogical beliefs with current thinking on teaching and learning mathematics and to increase their effectiveness in teaching mathematics. Further, researchers who tracked pre-service teachers into their first year of teaching noted that pre-service teachers' beliefs were challenged, resulting in some reverting back to traditional practices (Ensor, 2001; Steele, 2001.)

Beliefs Influence Practices

Wilkins (2008), consistent with Ernest (1989), found that content knowledge, attitudes, and beliefs are all related to teachers' instructional practices. Prior research on pre-service elementary mathematics teachers has mostly examined pedagogical beliefs in teaching mathematics, beliefs about one's own teaching ability, or content knowledge as individual constructs (Ball, 2003; Burton, 2006; Hart, 2004; Hill, et al, 2008; Morris, et al, 2009; Philipp, 2007; Swars, et al, 2007; Wilkins & Brand, 2004). The dilemma that continues is to understand the various changes in these constructs that take place over time after these teachers enter their own classroom.

Swars, et al (2007) noted that it is common for pre-service teachers to begin their professional teacher preparation programs with a more traditional view of what it means to know and teach mathematics whereas university mathematics education programs are more likely to promote the constructivist view of teaching and learning mathematics such that it is supported by the National Council of Teachers of Mathematics (2000). For pre-service teachers to be successful, they need to change their beliefs about mathematics education (Richardson & Placier, 2001). Stipek et al (2001) found that teachers who embraced traditional beliefs about mathematics and learning had lower self-confidence and enjoyed mathematics less than teachers who held more inquiry oriented beliefs. Further, more traditional beliefs were associated with more traditional practices. The findings indicated that teachers held a coherent set of beliefs, which predicted their instructional practices.

Teacher's beliefs about teaching and learning mathematics influence their practice (Hofer & Pintrich, 2002; Holt-Reynolds, 2000a; Muis, 2004; Scott, 2001). Tirosh (2000) states that there is a relationship between teachers' beliefs about mathematics, the use of manipulatives, and of the purposes for using manipulatives (Even & Tirosh, 1995; Tirosh, 2000). Beliefs orient actions, which in turn condition beliefs (Even & Tirosh, 1995; Tirosh, 2000). In their commentary on relations between policy and practice, Cohen and Ball (1990) suggest that teachers' pre-existing practices, beliefs, and knowledge affect changing classroom practices in relation to district policies. Beliefs influence teacher decision-making, practice and behavior (Wilson & Cooney, 2002) and can serve as a pre-cursor to real change in teaching practice.

Research has established that teacher beliefs about teaching and learning mathematics are linked to the instructional strategies they use. Therefore, those beliefs are linked also with student learning in the classroom (Phillip, 2007; Wilson & Cooney, 2002). Research has shown that beliefs develop over a period of time (Richardson, 1996). Beliefs develop over an individual's years as a student (Lortie, 1975) and are well-developed by the time a student enters college (Pajares, 1992). Wilcox, et al. (1991), found that pre-service teachers changed their beliefs about mathematics during their university experiences but not their beliefs about how the mathematics curriculum should be presented. More recent studies on change in the pedagogical beliefs and practices of pre-service teachers focus on realigning those beliefs and practices more closely with the previously discussed reform perspective on teaching and learning, looking at change over time (Hart, 2002; Lubinski & Otto, 2004; Wilkins & Brand, 2004).

Summary

Although existing studies connect the influence of content knowledge and teacher beliefs on classroom practices, there is a gap in the research that investigates how beginning teachers make use of concrete representations and how prior experiences and current teaching contexts impact teachers' use of concrete representations.

The literature on the constructivist perspective contends that constructivism is a stance toward learning where students utilize inquiry, discovery, representations, and discourse (Brooks & Brooks,1993; Hsueh, 2005). Learning is an active attempt of students to construct meaning from the things around them (Culatta, 2012). The teacher's responsibility is to understand the connection between how individuals learn and the instructional needs of the individuals, and then make decisions on the use of various strategies to best accommodate learning (Fosnot, 1996). In order for education to be effective, content and experiences must be presented in such a way that learners can connect new information to prior experiences, allowing them to construct their own understanding. Constructivist researchers (Bruner, 1966; Lesh, 1987) have contributed to the popularity of the use of mathematical representations.

Representations offer an opportunity for learners to connect real-life objects to more abstract concepts in order to make meaning and facilitate learning (Lesh 1987). Children often have difficulty with symbolic or abstract concepts, therefore they often need hands-on manipulatives and graphic pictorial representations of mathematical concepts. Lesh's model of representations (1987) identified five types of representation that occur in mathematics: real-life, manipulative, pictures, written, and verbal representations. Students move fluidly among and between the representations. Lesh contends that the ability of a learner to translate from one mode of representation to another is a means of investigating whether the learner understands a problem.

Research indicates that the proper use of manipulatives results in marked success in achievement, and manipulatives are particularly helpful in assisting students in understanding mathematical concepts (Anstrom, 2006; Baker, 2008). Further, the long-term, consistent use of manipulatives for mathematical instruction impacts student learning even more than sporadic use (Hartshorn & Boren, 2000). The impact of this research, which connects students' actions with physical objects to their mathematical learning, supports the use of manipulatives in the classroom (Boggan et al, 2010).

Over the past two decades, reform efforts have been made to improve student achievement in the United States. The National Council of Teachers of Mathematics (1999) developed a series of professional development standards in order to prepare teachers to address student shortcomings in the area of mathematics. In recent years, the Common Core State Standards (2010) have made efforts to equally align the requirements of students from one state to another across the country with achievement standards that are consistent and coherent from state to state. Attempts to extend reforms to teacher preparation programs have increased pre-service teachers' content knowledge and pedagogical practices by integrating course work with clinical experiences and extending clinical experiences (Cochran-Smith, 2003).

This case study considered the literature in seeking to investigate how beginning teachers make use of concrete representations for teaching mathematics and in what ways prior experiences and current teaching contexts impact beginning teachers' use of concrete representations. This study was designed to uncover patterns in how new teachers use manipulatives during instruction and what influenced those decisions. Should this study indicate a connection in one or more of these areas of investigation, the results may be used to inform the design of a larger scale study to investigate these influences in more detail.

Chapter III

RESEARCH METHODS

Research Questions

This dissertation describes a one year study of recent college graduates in their first three years of classroom teaching. The purpose of the study was to investigate the extent to which their pre-service instruction on the use of manipulatives and their current teaching context influenced manipulative use in their own classroom. The study focused primarily on two research questions: 1) How do beginning teachers make use of concrete representations for teaching mathematics; and 2) In what ways do prior experiences and current teaching contexts influence beginning teachers' use of concrete representations? Both qualitative and quantitative data were collected and analyzed in this study.

Research Design

In this research project, I used a mixed methods case study design (Dyson & Genishi, 2005; Miles & Huberman, 1994; Stake, 1995; Yin, 2003). The participants in this study were bound by the following characteristics: (a) teachers who graduated with a degree in elementary education; (b) graduated from the same university; and (c) have less than four years of teaching experience. In this context, I investigated how prior experiences (i.e. their own experiences with mathematics in school, their comfort level with the content knowledge, teacher preparation in manipulative use, etc). influences teaching practices. Because I believe that learning is socially constructed, is a lifelong process, is an act of reciprocity between and among a society of learners, and that teaching preparation is an evolutionary process, this qualitative study seeks to engage in a continued discourse and conduct a thematic analysis about mathematics instruction from the perspective of new teachers as learners. This chapter will discuss the process for data collection, beginning with the a) description of the participants, including criteria for selection and participant description; b) context of the study, including my role as a researcher; c) data collection strategies; and d) description of the design of the data analysis.

Participants

This case study consisted of three participants in order to collect in-depth data. Selection of cases was purposive (Miles and Huberman, 1994) in order to select participants that met specific criteria and were within a geographic proximity. The sampling allowed me to set the boundaries of the study by location and time limits of time. The following sections detail the section process and the selected participants.

Selection Process

A purposive criterion (Curtis et al, 2000; Schwandt, 1997) participant selection strategy was used. This study focused on the teaching of mathematics by new teachers who have graduated within the past four years from one southeastern university and who have access to concrete representations in their current school setting. I selected three participants based on a specific set of criteria and convenient access, selecting participants who were currently teaching at schools within a 30 minute driving radius of the university. To diversify the sample, I attempted to select new teachers who were a) currently teaching at different grade levels, b) in different schools, and c) preferably in different local school districts. Originally, I limited the number of participants to three so that data collection was thorough.

While completing their field experience where newly learned instructional strategies in teaching mathematics could be used and refined, these new teachers took a required mathematics methods course where manipulatives were introduced. Prior to mathematics methods, they had completed the courses Introduction to Discrete Mathematics, Basic Number Concepts for Teachers, and Geometry for Elementary Teachers with a grade of C or better. After mathematics methods, they completed one semester of full-time student teaching under the guidance of an experienced teacher mentor. These new teachers graduated with a GPA of 2.75 or higher within the past three years and had passed Praxis II and the PLT, therefore held current state teaching certification. At the time of the study, each participant was currently teaching in her own classroom on a full-time, permanent basis and had less than four years of teaching experience. Each teaches a different grade level in a different school, thus providing diversity among the participant contacts. Participants volunteered for the study.

After locating five potential participants whom I knew met these criteria and had jobs in local districts, I invited volunteers using a general e-mail request describing the study, detailing what would be required from them, and asking if they would be willing to participate. Initial contact was made in late September. Participants were told:

1) that participation was strictly voluntary,
2) they would be asked to video record some of their lessons,
3) recordings and all observational notes would be viewed only by me and anyone transcribing notes but would not be viewed by anyone else or made public in any way,

4) all conversations, lessons, and other materials collected would be strictly confidential and destroyed three years after results were collected

5) they would participate in two interviews:

   (a) the first interview would be a pre-lesson interview asking their perceptions about mathematics, their preparation to teach mathematics in their university and internship experiences, and how frequently they felt they use concrete representations in their instruction,

   (b) the second interview would be a post-lesson interview after all lessons had been recorded, viewed, and transcribed asking about specific instructional decisions made during instruction and what influenced those decisions,

6) upon completion of all interviews, they would be asked to participate in one focus group meeting with all the participants,

7) they were not offered a stipend,

8) I would try to keep their work load to a minimum,

9) they would receive a letter of recognition of participation in the study to include in their portfolio,

10) none of the participants would be in a classroom setting with me or under my employment during the study, therefore

11) an honest, unbiased responses from the participants was a reasonable expectation.

A letter of informed consent was delivered to participants, explaining the study and asking for their willingness to participate and explaining the risks and benefits of the study. This study would 1) not affect their employment record in their assigned teaching placement, and 2) not require additional preparation. Further, for the sake of anonymity, pseudonyms would be assigned to each participant. My contact information was included in order that participants with questions could contact me for answers.

Description of Selected Participants

Amy was an elementary major at a local university where I served as a mathematics teacher educator and had one semester of prior teaching in a second grade classroom at the onset of the study. Her placement at the time of the study, however, was her first full-year contract in a fourth grade classroom. I had supervised Amy in a field experience but had not taught her in any classes or supervised her internship. Her school was a Title I Reward school among the highest performing Title I schools in 2012 with 63% of students on free or reduced lunch located 3 miles from where I was centrally located. Based on the state Annual School Report Card Summary, the school had an Absolute rating of "Average." Overall scores in mathematics for 2012 were 32.2% exceeding expectations, 37.8% meeting expectations, and 30% not meeting expectations. Based on the ESEA/Federal Accountability Rating, the school received an overall grade conversion of "A" indicating its performance substantially exceeded the state's expectation in 2012. There were 20 students in her classroom throughout the school year.

Beth was an elementary major at the same local university and was beginning her first year of teaching in a fifth grade classroom at the onset of the study. I supervised her internship experience, but I had not taught her in a mathematics methods or any other course at the university. Her school was a Title I school with 74.3% of students on free or reduced lunch located in a rural area in a neighboring district and located 33 miles from where I was centrally located. Based on the state Annual School Report Card Summary, the school had an Absolute rating of "Average." Overall scores in mathematics for 2012 were 28.3% exceeding expectations, 42.1% meeting expectations, and 29.6% not meeting expectations. Based on the Elementary and Secondary Education Act (ESEA)/Federal Accountability Rating, the school received an overall grade conversion of "B" indicating its performance exceeded the state's expectation in 2012. She began with 16 students in her class but that number dropped to 15 by the end of the study.

Carly was an elementary major at the same local university and was beginning her third year of teaching in a second grade classroom at the onset of the study. I had not supervised Carly as an intern or taught her in mathematics methods, but I had taught her for two subsequent semesters in courses unrelated to mathematics. Her school was a fairly affluent school with 28% of students on free or reduced lunch located 6 miles from where I was centrally located. Based on the SC Annual School Report Card Summary, the school had an Absolute rating of "Excellent." Overall scores in mathematics for 2012 were 53.6% exceeding expectations, 33.5% meeting expectations, and 13% not meeting expectations. Based on the ESEA/Federal Accountability Rating, the school received an overall grade conversion of "A" indicating its performance substantially exceeded the state's expectation in 2012. There were 23 students in her class throughout the school year.

The fourth participant had to drop out of the study for medical reasons; therefore, her data were not included in the study.

Context

The setting of the interviews and observations was each new teacher's classroom using the curriculum that is adopted by the district which the new teacher teaches. I held the interviews in the classroom to promote comfort on the part of the participants and after school to ensure that enough time was allowed for a complete interview process with discussion and clarification. Observations of lessons and the video recordings were held in the classroom during the time regularly scheduled for mathematics instructions within the classroom, in order to ensure the most authentic environment for students and teacher. The focus group was held in a local restaurant and scheduled at the convenience of the participants.

My Role as the Researcher

The role of the researcher in a qualitative study is to acknowledge his/her beliefs and the roles his/her views may play in collecting and analyzing data. Fink (2000) argued that many problems in archiving data are due to the researcher's substantial role in the research process as he/she is personally involved in every step taken. Because of our prior experiences working together as they were undergraduates, all three participants knew my views in relation to the importance of using manipulatives for mathematics instruction. I did not want this to influence their responses. I expressed to participants that there was no right or wrong answer that the participants could reveal. I was simply trying to uncover the influences on new teachers' current mathematics instructional practices through a variety of means. *Why* do the new teachers in this study teach the way they do?

Data Collection

In order to triangulate findings, multiple data types were selected. To document the cases, I conducted six interviews (two for each participant), collected six weeks of video recorded lessons (two one-week "scoops" for each participant) (Borko, et al, 2005; Borko et al, 2007) and student artifacts, and facilitated one focus group meeting. There were two goals for the interviews. The first round of interviews related to beginning teachers' prior experiences in their formative years as well as their attitudes and beliefs relating to mathematics. The video recordings and student artifacts were to determine the actual amount of time each representation was used in mathematics instruction. The second round of interviews and the focus group concentrated more on their perceptions of the influences of their current teaching practices.

Gathering data from multiple sources helped triangulate data ensuring that what was seen and heard occurs in multiple instances and from multiple sources. In this way, findings were "cross validated" safeguarding the dependability of the study (Altheide & Johnson, 1994). Presentation of the data collection is described below in the order each occurred within the course of the academic year.

Initial Interviews

An initial audio-recorded interview was done with each participant on-site. The purpose of this interview was to understand a) how they felt about mathematics as they were growing up and experiencing instruction in mathematics from kindergarten through high school and college, b) whether they liked math, c) how proficient they felt they were in mathematics, and d) how mathematics was taught to them as they came through school. Participants were asked questions about their experiences in school mathematics, their beliefs about mathematics instruction, and their own teaching practices. The semi-structured interview protocol (Appendix A) afforded participants an opportunity to express their experiences in their own way while specifically addressing research question two.

The content of the interview also included participants' perceptions of their preparation at the university level in relation to content knowledge, their perceptions of the practices used in mathematics instruction by their university mathematics methods instructor in their pedagogical preparation, their perceptions of the practices used for mathematics classroom instruction by their mentor, and their perceptions of their current working environment.

Recorded Lessons

To address research question one, participants were asked to make video recordings of their instruction in mathematics. This was borrowed from Borko's (2005) "Scoop Notebook." Hilda Borko developed a system of artifact collection and scoring procedures that characterize classroom practices in science and mathematics. A significant part of this Scoop Notebook was the use of video-recording to take a "scoop", or snapshot, of what average classroom instruction was like on a day to day basis. I asked participants to record their mathematics instruction for five consecutive days in the fall semester (late October/early November). I selected that time frame because I wanted to make sure that instruction did not fall within the "holiday season" where activities and additional school programs were taking place. I wanted to get a true snapshot of what instruction looked like on an average day. These video-taped lessons were conducted in their natural setting of the participants' regular classroom with their own students. Participants were asked not to plan "special" lessons, since they knew they were being recorded. I wanted them to record five days of instruction that looked most like what they typically did. I asked them to do another "scoop" video recording during spring semester. I again asked participants to record their mathematics instruction for five consecutive days in late March/early April. I selected this time frame because I wanted to make sure that instruction did not fall at a time that was close to Spring Break, state testing, a holiday, or the end of the school year. Again, I wanted to get a true snapshot of what instruction looked like on an average day for one full week. At this time, participants again were asked not to plan anything "special" because they knew they were being recorded. I wanted them to record five days of instruction that looked most like what they typically did.

To facilitate this, I checked out a "Swivl" device that tracks the teacher's whereabouts in the room and has the microphone attached to the teacher. This allowed maximum opportunity to see what the teachers and the students in their proximity were doing and to hear what the participants and their students were saying. The Swivls, however, proved to be problematic for two of the participants. The first participant, although capturing quality pictures and audio, expressed frustration with trying to get it to work consistently and said that it shut off at random times. The second participant, although placing the camera in the back of the room in order to view the entire class at once and capturing a quality picture, accidently turned the microphone off not realizing that it came on automatically with the camera. This made hearing her comments and those of the students very difficult without the use of headphones and repeated play. I was, however, able to capture everything that she said and most of the children's responses (either by deciphering what they said or by inferring what they said based on the teacher response). The third participant had no difficulty with the Swivl and submitted recordings with high quality sound and picture. For the second video recording round, because the Swivls were frustrating for the participants, I gave them the option of using the Swivl or recording their lessons on the iPads. When they were completed, as I had done the first time, I gave the equipment to technology personnel to put them onto a disk prior to turning the equipment back in.

Second Interviews

The next phase of data collection was the participant post-video interviews. I reviewed the data from the video recordings coding (which will be discussed in the data analysis section) prior to meeting with each participant in order that the early information might guide some of my questions during the interview. The protocol for the post-video interview (Interview #2) can be found in Appendix B.

All interviews were audio-recorded. The first participant's scheduled interview was held on the university campus since the school was in a neighboring district and the participant lives near the campus. This interview took approximately 55 minutes. The second participant's scheduled interview was held in the classroom at the school where the participant teaches, which is in close proximity to the campus. The interview took approximately 50 minutes. The third participant's interview was held in the classroom at the school where the participant teaches, approximately 15-20 minutes away from campus. Due to scheduling conflicts, this interview was held a few hours before the focus group. This limited my opportunity to reflect at length on our discussion in preparation for the focus group. I did make notations of a few things from the interview that I wanted to open up to the group for discussion related to her comments. The interview took approximately 50 minutes.

Focus Group

In preparation for the focus group, I referred to my notes from the interviews to look for common themes that would be worth discussing. I also looked for things that I found interesting and possibly relevant in an interview mentioned by one or more participants but that I wanted to hear further discussion from and interaction between the group members. In order to prepare my questions, I formulated the questions for the focus group meeting based on participant responses and their practices using an opening question, introductory questions, transition questions, key questions, and ending questions (Morgan, 1997; Krueger, 1994). The protocol for the focus group questions is located in Appendix C.

All three participants and I met at a local restaurant for the focus group meeting. Participants were told that the interview would take approximately two hours, that I would buy them dinner, and that we would be discussing their mathematics instruction and the factors that influence their instructional choices. The focus group met to discuss their beliefs regarding mathematics instruction, their practices in teaching mathematics, and their concerns and possible roadblocks. I was aware that the dynamics of the focus group would be somewhat different from that of the individual interviews and observations; therefore, additional issues might arise that warrant further discussion and investigation. During this discussion, I probed into any patterns that arose in data that were already collected, as well as gathering further information about the participants' current teaching environment. I inquired about how participants felt their teaching environments have had an effect on their current practices. Like the interviews, this meeting was audio recorded. Participants were again assured that their interviews would not be heard by anyone but a possible transcriber and me. After the focus group met and responses were collected, I listened to the meeting several times and transcribed the conversation. I planned to use the transcriptions to conduct a thematic analysis relating to the collection of responses.

Data Analysis

Several data types were collected, therefore presentation of the analysis and results will mirror data collection. I will first summarize the process that was used and then break the analysis into individual means of data collection to discuss how the information was prepared. I will then present the analysis of each means of collection within the context of the discussion of the *group as a whole* and then by *participant*.

Initial Interview

The first steps for my analysis were transcribing, coding, and analyzing the initial interview with the participants. I used recordings of the interviews, which were conducted in the classroom setting. Information was organized by research question. This process allowed me to specifically address the first research question, "How do beginning teachers make use of concrete representations for teaching mathematics?"

The purpose of the first interview was to discuss participants' perceptions of their current teaching practices and their preparation to teach mathematics prior to entering their own classroom. When I had read all of the transcripts once, I re-read them making notes that related to their comfort level in understanding and teaching mathematical concepts, their perception of how often they use manipulatives for mathematics instruction in their current teaching setting, and their perception of their university and internship experience in relation to their exposure to and practice with mathematics manipulatives. I coded statements based on similar topics that arose. For example, statements such as "The professor was showing us how to teach the kids instead of having to focus on teaching us content," and "He did more lecturing versus hands-on, so we had to listen a lot and we didn't have as many practice problems as I would've liked," were coded as "coursework."

Statements like, “not daily but weekly when we are introducing a concept,” and “I have all those manipulatives over there but I just haven’t had time to look at them and see what’s there and what to do with them,” were coded “use of manipulatives.” The last sentence of the second statement was also coded “time.” Contents of interview transcripts, development of the coding process, and the results of these interviews will be discussed in greater detail in Chapter 4.

Recorded Lessons

The second step for my analysis was transcribing, coding, and analyzing the lesson video recordings, which were conducted in the classroom setting with their own students. This process also allowed me to specifically address the first research question, “How do beginning teachers make use of concrete representations for teaching mathematics?”

The video recordings of approximately 300 minutes with each participant teaching mathematics, their transcriptions, and coding were to determine the actual amount (versus participants’ perceived amount) that beginning teachers were using mathematics manipulatives and other forms of representation during their mathematics instruction. I will first describe my process for the development of coding and then discuss how that information was compiled and analyzed.

I set up the transcription into a word document table with the following categories: transcript number, time, text, code 1, and code 2. This table enabled me to have a format that would allow me to analyze the data using IBM SPSS Statistics. The transcript number was my code for the school, grade level, and lesson being observed. This column was relevant because it allowed me to separate each school from the whole data set in order to isolate the practices of each participant. The time column was broken into three-minute intervals and allowed me to identify specific context and sequence of events when relevant within the whole data set. It also gave me a frame of reference when comparing my coding to the coding of my colleagues. The text column included the activity that was occurring at each of the three minute time benchmarks. The Code 1 column indicated the form of representation being utilized (use of manipulatives, real-life situations, pictures, written symbols, or verbal discussion) and code 2 indicated who was doing the activity, the teacher or the student. Three coders independently coded the data and then compared codes. The codes were discussed and consolidated until consensus was reached. Table 3.1 shows a sample of the table.

Table 3.1 Video-Recorded Lesson Template Record Sheet

Transcript Number	**Time**	**Text**	**Code 1**	**Code 2**
F.5.1	3:00	Student is working on the Promethean board doing a multiplication problem.	WRI	S
F.5.1	6:00	Teacher explains what to do it you have 3 feet 6 inches.	VER	T
F.5.1	9:00	Student is answering a question (What is 48+2)	VER	S
F.5.1	12:00	Students are working with a partner to solve word problems converting feet to yards and recording answers.	WRI	S
F.5.1	15:00	Students are working with their partners working on solving problems converting lengths and recording answers.	WRI	S
F.5.1	18:00	Students are working with their partners to solve length conversions and recording answers.	WRI	S
F.5.1	21:00	Two partners are debating about the correct answer as they try to solve the length conversion problem.	VER	S
F.5.1	24:00	Teacher says, "Let's do a quick check to see where we are at." Begins questioning students about their responses and individual students respond	VER	S

Key: MAN-Concrete Manipulative, RL-Real-life, PIC-Pictures, WRI-Written, VER-Verbal

I wanted to look at the data from several perspectives. The first was an overview of the practices of all of the participants collectively. I specifically wanted to know overall among participants the percentage of the time manipulatives were being used for mathematics instruction, but I also wanted to see the percentage of time students and teachers were using pictures, real-life situations, written symbols, and engaging strictly in verbal interaction. I was particularly interested in how much of the time students were actively engaged versus how much of the time students were passive learners as the teacher talked and modeled.

In addition to looking at an overview of all participants, I wanted to analyze *each participant's* data separately from the whole group. Again, I was interested in the amount of time manipulatives were being used and the percentage of time pictures, real-life situations, written symbols, and verbal interaction were being used. I was also interested in the extent to which students were active in their use of various representations versus passively following teacher directions.

In order to summarize representation use, I constructed a frequency distribution to compare observed frequencies of the use of various representations that were collected through video observation for the whole group. Using SPSS, I constructed the frequency distribution for the entire data set for both teachers and students combined in relation to how often they used each of the five representations (Lesh et al, 1987). Then I examined only teachers in relation to how often they used each of the five representations. Finally, I examined only students in relation to how often they used each of the five representations.

Once I had done this for the entire group I constructed a frequency distribution in order to compare observed frequencies of the use of various representations used in *each participant's* classroom. Using SPSS, I constructed a frequency distribution of the first classroom for both teachers and students combined in relation to how often they used each of the five representations. Then I examined only teachers in relation to how often they used each of the five representations, followed by only students in relation to how often they used each of the five representations. I repeated this process for the second classroom and then the third.

In Chapter 4 the statistical data for all three participant video-recorded lessons will be presented as a whole, looking at each component of the frequency distribution for the total group. This will give the reader a sense of what is happening overall in terms of the use of various representations among the new teachers that participated in the study.

Post-Video Interviews

The next step for my analysis was transcribing post-video interviews, which were conducted in the classroom setting, in order to identify themes that arose during my one-on-one discussions with participants. This allowed me to specifically address the second research question "In what ways do prior experiences and current teaching contexts influence beginning teachers' use of concrete representations in their mathematics instruction?"

Once I had transcribed all of the interviews, I categorized statements as they related to each research question, then looked for themes within each. I did not go back and look for final themes until I had completed the transcription of the focus group meeting.

When I had read all of the transcripts without notes once, I re-read them making notes as I went through that related to recurring themes. I read through a third time making notes and using color coded highlighting for themes. The themes that emerged from the second interviews were technology, Common Core, the textbook and related materials, behavior, time, professional development, and the teaching teams. These findings are discussed in detail in Chapter 4.

Focus Group

The final step for my analysis was transcribing the focus group discussion. The focus group meeting was held in a restaurant for ease and flow of discussion. It explored possible themes that arose during my one-on-one discussions with participants and helped identify common themes and concerns within and among the group of participants. The data from the focus group allowed me to further address the second research question "In what ways do prior experiences and current teaching contexts influence beginning teachers' use of concrete representations in their mathematics instruction?"

After completing the transcription of the focus group meeting, I reread the transcription without any marking. As was the case with the interview transcriptions, I wanted to get the feel for the flow of the focus group discussion. Once I read the transcription of the focus group in its entirety, I went through highlighting in one color all of the statements that seemed to address the question. I formulated ideas about which statements could be grouped into categories. I began separating highlighted themes into different categories, giving each one a different color. Once the results for each participant are revealed, the initial interview, the post-video interview, and the focus group meeting will be discussed at length in relation to what participants perceive to be the factors that have influenced and are influencing their teaching practices in mathematics.

Conclusion

Chapter 3 described the research methods used to generate and analyze information to answer the research questions: 1) How do beginning teachers make use of concrete representations for teaching mathematics; and 2) In what ways do prior experiences and current teaching contexts influence beginning teachers use of concrete representations? I discussed the selection of participants, the data collection, the instruments used to collect the data, and data analysis.

Quantitative data analysis involved coding video-recordings and then logging instances of representation use. Qualitative data analysis involved a thematic analysis of interview and focus group transcripts that afforded participants the opportunity to discuss their instructional practices in mathematics and the influences of those practices. Results of the data analysis are presented in Chapter 4.

Chapter IV

Results

In this study, I ask two primary questions: (a) How do beginning teachers make use of concrete representations for teaching mathematics; and (b) In what ways do prior experiences and current teaching contexts impact beginning teachers' use of concrete representations? To address the first question, I video recorded beginning teachers' mathematics instruction for a two week period, coding the representation types as discussed by Lesh et al. (1987), which were apparent during video clips of instruction. I ran a frequency distribution to determine the percentage of the use of each representation, and conducted interviews relating to their instructional decisions. To address the second question, I conducted interviews and a focus group with beginning teachers to discuss the instructional practices of each participant, the variation in instructional practices among participants, and each participant's perception of what influences her instructional practices. Using an inductive coding scheme, I put similar pieces together, assembled them into "chunks", and identified any linking pieces so that the big chunks could be tied together (LeCompte, 2000). I then coded the data in order to organize the information into categories so that the data could be organized and verified, the findings reported, and implications of the study discussed (Lincoln & Guba, 1985; Miles & Huberman, 1994).

This chapter presents results in two parts in order to discuss the two primary questions of the study. The first part will address, "How do beginning teachers make use of concrete representations for teaching mathematics?" The data are presented in tables and relevant observer notes from video observations, with a brief narrative to explain how the data in the table were used. In some instances, participant interviews and focus group discussion supported the findings from the video recordings; however, in some instances a participant's perception of her use of manipulatives varied significantly from her actual usage. The purpose of this portion was to determine what types of representations were used in the classroom and to what extent each was used.

The second part of Chapter 4 will address the second question, "In what ways do prior experiences and current teaching contexts impact beginning teachers' use of concrete representations?" I will present a qualitative examination of the interviews and the focus group to discuss each new teacher's practices and perceptions of the influences of those teaching practices. Finally, I will summarize the overall themes that reoccurred during my discussions with all three participants to attempt to identify perceived influences of manipulative use among beginning teachers. In order to preserve the identity of the participants, each participant was given a pseudonym: Amy, Beth, and Carly. A description of each of the participants is included in Chapter 3.

Part 1

Examining the Question "How do beginning teachers make use of concrete representations for teaching mathematics?"

A variety of representations were apparent in the videos, such as the use of pictures on the Promethean board, explanations, equations, and the use of various measurement tools. Table 4.1 shows examples of different events using a variety of representations and how they were coded.

Table 4.1 Representation codes

Event	Code 1	Code 2
Student is working on the Promethean board doing a multiplication problem.	WRI	S
Student is answering a question (What is 48+2)	VER	S
Teacher confirms that an answer is correct. She reads the next problem.	VER	T
Tommy is answering the question, “How many times does three go into twelve?”	VER	S
Using the Promethean board, the teacher models how to measure to the half inch and quarter inch using a picture of a ruler.	PIC	T
Using the ruler on the Promethean board teacher poses the question how long an object is (to the quarter) and points to the ruler.	PIC	T
Students are using a ruler to measure a paper clip.	MAN	S
Students are talking in partners about how they can tell the difference between an inch and a half inch when measuring with a ruler.	VER	S
Teacher is demonstrating on the Promethean board (picture of a ruler) as a student describes how to read the ruler	PIC	T
Students were explaining their reasoning on how to determine the differences in measuring with the ruler (inch, half inch, quarter inch).	VER	S
Teacher is showing the ruler on the Promethean board and demonstrating the whole, half, and quarter inch.	PIC	T
Students are working in assigned partners measuring items in the practice set.	MAN	S
Teacher says, “Yesterday we talked about measurement. What tool do we use when we are measuring?” Students respond.	VER	S
Students are working in partners on p 789 Am I Ready?” questions to review (multiplying) and writing their responses.	WRI	S
Students from groups are coming up to the Promethean board to do (write) the problems that they solved.	WRI	S
Student is explaining step by step how they solved their multiplication problem.	VER	S
Teacher is writing (and explaining) how to multiply a problem on the Promethean board.	WRI	T
Teacher is explaining how to solve the multiplication problem and “trial and error”.	VER	T

Key: MAN-Manipulatives, RL-Real-life, PIC-Pictures, WRI-Written, VER-Verbal

Data from the overall video lesson transcriptions indicate that manipulatives were used significantly less than other forms of representations. Table 4.2 shows the percentage of time each representation was used.

Based on videotaped observations, manipulatives were used in 38 distinct times at 12.4% of instructional time; pictorial representations were used in 74 actual observations at 24.1% of instructional time; written expression was used in 57 actual observations at 18.6% of instructional time; and verbal communication was used in 138 actual observations at 45% of instructional time.

Table 4.2 Instruction method percentage of whole group by teachers and students

		Frequency	Percent
Valid	Manipulatives	38	12.4
	Pictures	74	24.1
	Written	57	18.6
	Verbal	138	45.0
	Total	307	100.0

As I examined the events and how they were coded, I wanted to differentiate when manipulatives were being modeled by the teacher or whether the students were engaged by manipulative use. Some of the coded events which included teacher use of manipulatives were:

1) Teacher says, "At your tables I set down three hexagons. Pick them up and pass them around. So each one of these weighs about a gram. So if you wanted to measure a kilogram, it would take 1000 of these." (Amy, Lesson 4)
2) Teacher says, "Everybody hold up 7 fingers. Let's do the problem that Haley just gave us seven minus three. Everybody hold up seven fingers." Students do. "Take four of those fingers away." (Carly, Lesson 3)
3) Teacher is demonstrating how to close the tape measure, pressing the black button (so that it doesn't hit anyone in the face). (Carly, Lesson 7)

I noticed that in most of the events coding teachers using manipulatives, the use of concrete representation was accompanied by explanation. In contrast, many of the events which included student use of manipulatives were either strictly the action or the action accompanied by verbal representation that was initiated by the teacher or written representation:

1) Students are using a ruler to measure a paper clip. (Beth, Lesson 2)
2) Students are using rulers to measure the length of items around the room in millimeters, centimeters, and meters. (Amy, Lesson 1)
3) A student is using a ruler to measure a window. She is recording the measurement onto her paper. (Carly, Lesson 6)
4) Students are finding two things at their seat that they can measure in inches to see how many inches it is. (Amy, Lesson 7)
5) A student is measuring in inches with the tape measure. Teacher asks "So what is it?" Student responds "48 inches." (Carly, Lesson 7)
6) Student measured her cubby and explained how she did it earlier. (Carly, Lesson 7)

The statements above exemplify cases in which manipulatives were used and verbal representations were used either by a student or teacher. In other words, there was verbal representation, but the use of manipulatives outweighed the use of verbal representation in isolation, earning a code for the use of manipulatives. All of the 38 statements coded for manipulatives related to measurement with the exception of the following six:

1) Logan answers "6" when asked how many faces a cube has. Teacher gets a base ten thousands cube and shows it to the class. Together they count the faces. (Beth, Lesson 6)
2) Students are cutting out nets and folding them into three dimensional shapes. (Beth, Lesson 6)
3) Tommy took a net and folded it to make it into a cube. (Beth, Lesson 6)
4) Students are working in groups to solve problems on volume. They are writing their responses in their workbook. Once they have their answer, they must verify their answer using the cubes. (Beth, Lesson 8)
5) Students have disassembled their blocks and are looking only at layer one. A Student explains that all of the layers are equal because they all have the same number in them and are all alike. (Beth, Lesson 8)
6) Speaking to an individual student, teacher has student using snap cubes to create volume to solve the problem. "Stack them four across and four up tall." (Beth, Lesson 8)

When looking at the total time, the majority of instructional time is spent on verbal communication, followed by pictures then written communication, whereas the use of manipulatives is last. All of these percentages refer to representations used by either teacher or students. This decision is based upon my review of the literature, which indicates that not only is the use of manipulatives an important representation for students understanding of mathematics, but also the extent to which students are engaged in their use versus the teachers' use of modeling and directing the use of manipulatives (Anstrom, 2006, Baker, 2008; Boggan et al., 2010; Hartshorn & Boren, 2000).

After looking at the overall frequency distribution, I wanted to examine the differences in activity specifically by teachers, and then specifically by students. Table 4.3 shows the breakdown of use by teacher participants' use and then separately by students' use of a given representation. The "primary agent" is the primary person(s) using one of the four representations.

Table 4.3 Examining differences in activity by teachers and students

Primary agent * Instruction Method Cross Tabulation							
			Instruction method				Total
			Manipulative	Picture	Written	Verbal	
primary agent	Teacher	Count	4	28	12	79	123
		% w/in primary agent	3.3%	22.8%	9.8%	64.2%	100.0%
		% w/in Instruction method	10.5%	37.8%	21.1%	57.2%	40.1%
		% of Total	1.3%	9.1%	3.9%	25.7%	40.1%
	Student	Count	34_a	46_b	$45_{a, b}$	59_c	184
		% within primary agent	18.5%	25.0%	24.5%	32.1%	100.0%
		% within Instruction method	89.5%	62.2%	78.9%	42.8%	59.9%
		% of Total	11.1%	15.0%	14.7%	19.2%	59.9%
Total		Count	38	74	57	138	307
		% within primary agent	12.4%	24.1%	18.6%	45.0%	100.0%
		% within Instruction method	100.0%	100.0%	100.0%	100.0%	100.0%
		% of Total	12.4%	24.1%	18.6%	45.0%	100.0%

The challenge was, given that there were four different uses of representations, to determine where the differences were occurring between primary agents' (teachers' and students') use of representations. When examining the observations specifically for each of the forms of representation (instruction method) and their use during instructional time, teachers used manipulatives 10.5% whereas students used them 89.5% of the time. Teachers used pictorial representations 37.8% of instructional time whereas students used them 62.2%. Teachers used written expression 21.1% of instructional time whereas students used it 78.9%. The use of verbal expression had a smaller yet still significant gap between the two with teachers using verbal expression 57.2% of instructional time and students using it 42.8%. With all four representations, students used each more frequently than teachers, indicating a more active than passive learning experience. This will be examined further in the discussion section.

Looking at the overall total for each type of representation among all three participants it is apparent that all three participants relied heavily on verbal expression as their primary means of instruction. Although manipulatives were used 12.4% of instructional time, pictorial representations were used 24.1%, and written expression was used 18.6%, the use of these representations was often accompanied by explanation. More significant, however, is the percentage of time (45%) that verbal representation was used *without* the benefit of any other representation. *(Note *As was described in the last chapter, there is an assumption of verbal explanation in conjunction with the use of other representations. Recall that the code of **verbal** representation for an event was to be used only in the **absence** of any other form of representation.)*

For example, the following statements were coded "verbal," although an opportunity for additional representation was present but missed:

1) Two partners are debating about the correct answer as they try to solve the length conversion problem. *(ruler?)* (Beth, Lesson 1)
2) Student answers which measurement (200g or 200kg) they selected for an item and explained their reasoning. *(weights?)* (Amy, Lesson 4)
3) Teacher is asking how many pints are in a gallon. Students respond with random guesses. Teacher says "There's 8." *(measuring cups?)* (Amy, Lesson 5)
4) Different students are describing what they think math problems with 'doubles' are. *(solid or pictorial sets?)* (Carly, Lesson 4)
5) Teacher explained how the lines on the football field were called yard lines and that they were marked by how many yards from the end zone they are. *(yardstick?)* (Carly, Lesson 7)
6) Teacher is trying to explain to Luna that there are twelve inches in a ruler, or in a foot. Student is confused. *(ruler?)* (Carly, Lesson 7)
7) Student is explaining how he found the perimeter of a square. I knew it was 9 cm on this side and all the sides were the same so I multiplied 9x4 because there are four sides. *(sketch?)* (Amy, Lesson 6)

8) Bobby is giving his definition of a vertices. He said that they are the edges and there are four to a square. Taryn said the vertices are where the edges come together to meet. Teacher said, "So the vertices are like a corner. *(3-D shapes?)* (Beth, Lesson 7)
9) Teacher is explaining how to use subtraction to find the differences in length. *(snap cubes? rulers?)* (Carly, Lesson 8)

Individual Analysis

After looking at the overall data relating to all study participants, I wanted to look at each individual participant and her students in relation to the collective in order to examine the use of representations of each participant in order to accurately investigate the perceived influences of practices by specific individuals. Table 4.4 breaks down the data into the setting for individual participants. It also allows for a comparison of participants' use of each type of representation. Percent within school refers to what representations a teacher is using within her own classroom, which totals 100% of her instructional time. Percent within instruction method refers to each of the percent of time each teachers uses each of the representations (i.e. Amy uses manipulatives 15.8%, Beth 31.6%, and Carly 52.6% totaling 100% of the instructional time that all teachers used manipulatives.

Table 4.4 Examining differences in activity by school.

School * Instruction Method Cross Tabulation							
			Instruction method				Total
			Manipulatives	Pictures	Written	Verbal	
School	Amy	Count	6_{a}	28_{a}	23_{a}	52_{a}	109
		% w/in School	5.5%	25.7%	21.1%	47.7%	100.0%
		% w/in Instruction method	15.8%	37.8%	40.4%	37.7%	35.5%
		% of Total	2.0%	9.1%	7.5%	16.9%	35.5%
	Beth	Count	12_{a}	27_{a}	17_{a}	47_{a}	103
		% w/in School	11.7%	26.2%	16.5%	45.6%	100.0%
		% w/in Instruction method	31.6%	36.5%	29.8%	34.1%	33.6%
		% of Total	3.9%	8.8%	5.5%	15.3%	33.6%
	Carly	Count	20_{a}	19_{b}	$17_{a, b}$	39_{b}	95
		% w/in School	21.1%	20.0%	17.9%	41.1%	100.0%
		% w/in Instruction method	52.6%	25.7%	29.8%	28.3%	30.9%
		% of Total	6.5%	6.2%	5.5%	12.7%	30.9%
Total		Count	38	74	57	138	307
		% w/in School	12.4%	24.1%	18.6%	45.0%	100.0%
		% w/in Instruction method	100.0%	100.0%	100.0%	100.0%	100.0%
		% of Total	12.4%	24.1%	18.6%	45.0%	100.0%

Amy

Amy perceived the amount of instructional time spent using manipulatives to be "not daily but weekly when we are introducing a concept." Table 4.4 displays, among individual participants' manipulatives were used least often by Amy. She and her students used manipulatives only 5.5% of the observed instructional time and all six instances were with the use of rulers. Although pictures at 25.7% of instructional time were used significantly more than manipulatives, they were used most frequently in the form of charts for the students to copy rather than pictorial representations of mathematical equations or concrete objects. Written expression at 21.1% of instructional time was largely copying the charts from the Promethean board as opposed to mathematical problem solving. Verbal expression was used most frequently at 47.7%. During interviews, Amy noted her regular use of manipulatives when introducing a topic. This was inconsistent with findings from video recordings of classroom practice.

Although Amy felt that she had a good understanding of how to use math manipulatives based on her pre-service experience, she did not use manipulatives in her video-recorded instruction with the exception of rulers and protractors. She did use technological versions of manipulatives on occasion (virtual fraction circles/squares and a Geoboard app) and MobyMax for skill practice and assessment. The majority of my observations of her instruction were formula and textbook-based. Although the materials were not from a physical textbook, the content from the text was used through other media. Amy relied heavily on memorization of steps and formulas for her instruction.

> If there was something in the book that I liked, I would pull it and put it up on the (Promethean) board.
>
> I took conversion charts from the textbooks that we used to use that are discontinued and put them on the Promethean board.
>
> I mostly used the manipulatives just to get them to understand how much 12 inches would be? How much is a foot? How much is a meter?
>
> They did better with learning a formula…knowing that if you are going from small to big you divide and the other way you multiply.

Amy used technology as an integral part of her instruction. She frequently used the Promethean board to post charts from the text for the students to copy. She and her students used the iPads on a regular basis using the internet and various apps. She utilized MobyMax (an electronic curriculum for K-8 mathematics that can monitor student progress) and she used Edmodo and virtual manipulatives for instruction. Consequently, as

Amy discussed her use of manipulatives, she often discussed her use of a virtual manipulative or app rather than concrete materials. The following quote is indicative:

> I consider iPads to be a manipulative. But they also have a thing called MobyMax and, not only do they have a skills check, but I can put them at their own level.
>
> We found an app called virtual manipulatives so we did most of our fraction stuff using the app because they could take it apart and use it in squares and circles and they could compare the fractions and compare the decimals.
>
> I would introduce (a skill), then I always gave them a video 3-5 minutes to watch. I would put it on Edmodo so that they could go back and watch it again if they needed.

Beth

Based on the initial interview, Beth stated she used manipulatives in roughly seventy percent of mathematics lessons. Beth fell in the middle range of her use of manipulatives. She and her students used manipulatives 11.7% of the observed instructional time, although manipulatives were available and visible in the back of the room. When she did use manipulatives, she used rulers for linear measurement activities, nets that students folded into 3-D shapes in order to see the transformation from two dimensional to three dimensional, and snap cubes when teaching calculation of volume in 3-D shapes. Pictures were used 26.2% of instructional time and were noted in the form of pictures of rulers and pictures of nets to accompany the lessons using the manipulatives. Written expression at 16.5% was frequently used for solving written problems in the textbook. Like Amy, Beth and her class also used verbal expression most often at 45.6% of instructional time. During interviews, Beth noted that she used manipulatives roughly 70% of her instructional time which was inconsistent with findings from video recordings of classroom practice.

In her interview Beth reported that she had ample access to manipulatives that came with the series, but she did not use many of them in her instruction. This statement was supported in the video recorded lessons. She used the Promethium board consistently to display pages from the book and occasionally accompanying pictures. The majority of my observations of her instruction were textbook-led. Although Beth reports being strong in a variety of uses of technology, she did not use any apps or iPads in the instruction that I observed. Reasons Beth provided for not using technology will be discussed in Part Two of this chapter.

Carly

The participant who used manipulatives most frequently was Carly. She perceived the amount of instructional time spent using manipulatives to be about three out of five days. According to data from the video recorded lessons, she and her students used manipulatives 21.1% of the observed instructional time. Noted manipulatives included fingers, rulers, 3-D shapes, and tape measures. Among the three participants, she and her students used manipulatives significantly more often at 52.6% of the total amount of time using manipulatives (while Amy used them only 15.8% of the time and Beth used them 31.6%). Pictures were used 20% of instructional time, including dots and stars for counting, circled sets, “x”, and line plots. Written expression was used 17.9%, which was fairly in line with the other two participants. Verbal expression was used more than other representations at 41.1%, however this was the least of the three participants. Although she also relied heavily on verbal expression, the use of manipulatives came in second. During interviews, Carly estimated her use of manipulatives for instruction to be more than half of her instructional time, which was more consistent with findings from video recordings of classroom practice than her counterparts. In her interview, Carly rated herself above average in the use of manipulatives citing examples such as 3D shapes, counters, use of ladybug clocks, and measurement tools. However she reported that she did not use them much during the second half of the year.

Collective Comparison of the Use of Manipulatives

There was significant variation in the amount that manipulatives were used and only a slight variation in the types of manipulatives that were used. All three reported using rulers for measurement, and their recordings were consistent with those statements. In Amy's class, rulers were the only manipulative that she and the students used during the recorded lessons. Both Amy and Beth used the Promethean board to display pages from the book. In addition to rulers Beth and her students used nets that were folded into 3-D shapes and multi-link cubes. In addition to using rulers and 3-D shapes, Carly and her students used tape measures and had the students adding and subtracting using their fingers. Of the three participants, Carly used manipulatives the most.

Part 2

Examining the Question "In what ways do prior experiences and current teaching contexts impact beginning teachers' use of concrete representations?"

Investigating my second research question of "In what ways do prior experiences and current teaching contexts impact beginning teachers' use of concrete representations?" I focused on each participant's use of manipulatives. Examining the frequency of the use of manipulatives for each participant allowed me to use the interviews and focus group to investigate the perceived influences on those teaching practices.

In order to develop an understanding of what perceived influences impacted new teachers in their instructional practices when teaching mathematics, I analyzed our discussions during interviews looking for themes within each individual interview and commonalities and differences among and between participants. In this section, I will briefly discuss each participant individually in terms of her perceptions of her 1) knowledge base in mathematics, 2) pre-service experiences in mathematics, and 3) access to manipulatives in their current teaching context. I intertwined them individually which informed the collective interview that follows. I will then share a thematic analysis (Boyatzis, R.E. ,1998) of each participant's instructional practices and the perceived influences on her instructional decision-making. Finally, I will conclude with a collective thematic analysis of the focus group discussion. (*The demographics for each participant can be referenced under the "Participants" section of Chapter Three.*)

Amy

Based on discussion from our initial interview, Amy was confident in her knowledge of mathematics for teaching. She rated herself high on her scores on the SAT and ACT in mathematics, in how well she did in her college core math classes, and average or above average in the mathematical content areas in which she was questioned. She indicated that she had had extensive exposure to math manipulatives in her methods class.

> (The methods class) was using manipulatives and working with the children, so the time frame was better. I was exposed to base ten blocks, pattern blocks, attribute blocks, rulers, and yard/meter sticks in class and rulers and yard/meter sticks in my field experience and internship. I don't remember using Geoboards in class, but we did have the opportunity to use them in my field experience. I don't remember using fraction pieces, but we did activities to understand fractions, like paper folding (Amy, personal communication, October, 2013).

She felt that she had ample access to multiple varieties of manipulatives and in her current teaching setting because the last teacher had left many things. She felt she had a good understanding in the use of manipulatives because she had learned how to use them in her university methods class.

Impacting her decision to use the technology instead of the physical manipulatives were behavioral issues. Amy reported several times that students could not work well together in groups and often not even in partners.

> This group of kids didn't really work well in groups. They had trouble with sharing … We tried all year, and it didn't matter what we tried to do. They cannot handle being in small groups.
>
> With the geometric shapes, even after giving them a little time to play around with it, and several warnings that they shouldn't be building with it, I still had five or six that had to put them away because they couldn't handle it (Amy, personal communication, May, 2014).

Technology was used frequently. She chose to use the virtual fractions on the iPad because students did not have enough fractions pieces to have their own set. This meant they had to work in groups, which was problematic due to the behavior issues. In order to manage her classroom, Amy felt that allowing each student to work with the virtual manipulatives on their own iPad would enable each student to create each fraction that they needed without conflict with another student. Similarly, she opted for the Geoboard app so that students would not have access to the rubber bands. On occasion, she used the Promethean board to show students a 3-5 minute video to show them how to do a skill. In several instances, she used a chart for students to copy into their journals. I noticed this practice in several video observations, so I wanted to investigate further. When I asked her why she made the decision to have students copy the charts, she noted two reasons. One reason was so that students had something to look back at in their journal when doing conversions, and the other was in order to manage the behavior in the classroom by having them copy independently.

Amy also reported that this was the first year the school had implemented Common Core State Standards (CCSSM), therefore all of the teachers were still trying to learn these standards. She said they had training last year for CCSSM, but she was not familiar with the extent of that training. She reported that as her grade level team studied the curriculum, they found that it was very vague and that there were several gaps in the curriculum. The team met once a week to determine which standards they planned to address the following week and pull from online resources from other professionals who had already been doing CCSSM. They relied heavily on the textbook for the first half of the year but began to use it more as a resource during the second half of the year. The unfamiliarity with the new CCSSM and concerns for testing impacted her teaching practices as well as those of her colleagues.

Amy used formulas and repeated practice, sometimes posting pages from the textbook on the board and sometimes using other sources. Another issue relating to CCSSM was that teachers were concerned that the state test did not address CCSSM therefore the team felt that they needed to address both the state standards from the previous year as well as the current CCSSM.

> This is our first year of Common Core so we were trying to learn Common Core, too, and the depth that we need to go to. So when we looked at the Common Core standards and looked at the book, we thought that there were gaps (Amy, personal communication, May, 2014).
>
> I know last year they had some training for Common Core, but I graduated in December so anything that they had in the spring, I was

> technically a long-term sub, so I didn't do any of the professional development stuff. I know with Oxton, we used the SC standards so I was not very familiar with Common Core (Amy, personal communication, October, 2013).

> We would meet on Wednesdays and look at the Standards that we were going to address next week and we would pull from what we already had and from online resources where there were people who had been doing Common Core (Amy, personal communication, May, 2014).

> At the beginning of the year we were trying so hard to be aligned with the Common Core. It was a lot more vague (Amy, personal communication, May, 2014).

> I went back and taught them that stuff because they still were going to need to know how to do it on PASS because PASS isn't Common Core. So Common Core kind of gives you, not less stuff, but you have to know it more in-depth. Common Core is more like analyzing than explaining (Amy, personal communication, May, 2014).

When asked about staff development Amy reported that there were no in-service training sessions on how to effectively use manipulatives in the classroom. The district was pushing for more use of technology in the classroom, so most of the in-service trainings were focused on the use of technology and how to use the new textbook in order to address CCSSM.

> We didn't have specific in-service on manipulatives. I was on Team Math this year but it wasn't manipulative-based (Amy, personal communication, May, 2014).

The district's math coordinator came to a meeting to clarify some issues in terms of how the textbook should be used. Amy reported being a representative for her school on "Team Math," which was compiled of two teacher representatives from each school and met quarterly. These monthly meetings were held at the district office and representatives would return to their school and send an e-mail with the contents of the meeting to the teachers. None of these professional development meetings were presented to specific grade levels or the faculty as a whole.

> Team Math is something that helps us support (math instruction). Over the summer we went to a couple of trainings that was about the adoption of the new Investigations book (Amy, personal communication, May, 2014).
>
> We had concerns at the beginning of the year because (the series) was leaving out a lot of stuff (The coordinator) came to the school and told us that we could just use it as a resource. Over the summer, we had thought that we had to use this book and nothing else and it would be fine. But it ended up being that we had to pull other stuff, so she said it was fine to use it as a resource (Amy, personal communication, May, 2014).
>
> Team Math met four times and there were two representatives from each school. They mainly focused on how to integrate technology into what we were already doing. Then we would come back and share with the faculty what we had learned. We would just do e-mails and we would take turns typing it out and sending it. If teachers had questions, they could come to us (Amy, personal communication, May, 2014).

When asked how often teachers came with questions after her e-mail was sent, Amy responded that no one seemed to have any questions.

Amy reported in the focus group feeling that her university methods class and the prepared notebook of activities from that class had the greatest impact on her teaching practices. She stated that it was much more helpful than the district meetings. This statement was in contradiction to my observations of the lessons, where I observed only one activity from her methods class. It was also inconsistent with her statements during our interviews and the focus group.

Beth

In terms of her content knowledge in mathematics Beth felt that she had an average knowledge base for teaching mathematics, explaining that math was her weakness. When asked about specific concepts in mathematics, Beth rated herself slightly above average in probability, “not so good at fractions,” “pretty good at decimals,” strong in algebraic concepts, but fairly low in geometry. She reported feeling strong in measurement and very strong in place value and elapsed time. She reported that she felt good about teaching nets and three-dimensional shapes. She did well on standardized tests and in her core math classes at university with the exception of one that was all lecture.

> I felt that (my core math classes) were a good start to math, but that they were a little vague with teaching education to the elementary students (Beth, personal communication, October, 2013).
>
> I think my difficulties had to do with my professor’s way of teaching it. He did more lecturing versus hands-on, so we had to listen a lot and we didn’t have as many practice problems as I would’ve liked, which is something that I try to incorporate now, more practice and hands-on and moving around and doing math (Beth, personal communication, October, 2013).

When asked about her math methods course and pre-service experiences, Beth gave her university professor above average ratings for her use of manipulatives in her elementary math methods class, commenting that she had been exposed to base ten blocks, fraction pieces, pattern blocks, attribute blocks, rulers, virtual Geoboards, and yard/meter sticks. She said about her math methods class:

> I really enjoyed the strategies and the different methods that we learned and the activities were fantastic (Beth, personal communication, October, 2013).

She felt that she had ample access to manipulatives, referring to boxes of manipulatives in the back of the room that had remained unopened, at the mid-point of first semester. She said that she didn’t use them because the students weren’t accustomed to using them, there had been no training in how they should be used, and because she did not have time to go through them or explore them and figure them out. She did report sometimes using base ten blocks, meter sticks/rulers, and also using technological versions of manipulatives (virtual Geoboards). She used more pictorial representations (i.e. pictures from the textbook projected onto the board, paper fraction strips, online ruler, nets). In relation to her use of manipulatives Beth reported:

> I have all those manipulatives over there (points to two boxes), but quite frankly I haven't even opened them up. I just haven't had time to look at them and see what's there and what to do with them. We just got those materials with the series but that's it. The kids aren't used to it (Beth, personal communication, October, 2013).
>
> I didn't use (manipulatives) a lot. None of the fifth grade teachers did because we really didn't have instruction on how to use them. The online part of the book has virtual manipulatives, so I did use those. We used the virtual base ten blocks a lot and we used the (virtual) fraction bars (Beth, personal communication, May, 2014).
>
> We did do paper nets. I felt like the nets was important to use because kids can see the two dimensional figures change to three-dimensional and how you go from a three-dimensional to two dimensional when you unfold it (Beth, personal communication, May, 2014).

It was her understanding that the district wanted them to use the textbook as much as possible. She and the other teachers did use the book as their primary source, however they "tweaked" the order when they felt there was a need.

> We use a required series here "My Math" which is based on the Core Curriculum. But we don't use the Core Curriculum. We use the state standards. I use the book as much as possible and then I go to different sources (Beth, personal communication, May, 2014).
>
> I really liked the book itself. It was user friendly. You could tear out pages. There were quizzes. The online thing did a spiral deal where we could quiz them continuously on old stuff or we could bring in new stuff. There are a pre-assessments and post-assessments. There was vocabulary. It was good (Beth, personal communication, May, 2014).

> I just followed the curriculum map and found where it went in the book….so we kind of jumped around a lot where it was in the book (Beth, personal communication, May, 2014).

> The book provides a lot of problem solving. In fact every chapter has at least one problem solving lesson. And a lot of them will say, "Draw your visual here" or "Here's your draw box" (Beth, personal communication, May, 2014).

Beth did report using technology whenever possible in her classroom, but stated that they were limited in the number of computers that were available to students and the availability of internet. She relied heavily on the textbook saying that it was user friendly. She ultimately followed the curriculum map and plugged in instruction for specific standards where it could be found in the textbook. When possible, she used the online component of the series, posting pages from the text onto the Promethean board. She felt posting pages from the textbook made the lessons more interactive. Most of the activities that students completed were from activities in the book, including items entitled "real-life problem solving" and "draw your visual here".

> We have iPads, but we have to check them out from the library. We have some new tablets, but we don't have Wi-Fi (Beth, personal communication, May, 2014).

> Students can log on in the computer lab and teachers can log on. But for the teacher, it is the book but put online in slides. So you pull it up to your Promethean board so you can put up one answer at a time or you can pull them all up. It's very interactive, so I can have one student come up to the board and work it out on the board (Beth, personal communication, May, 2014).

> The book was required to use as much as possible, so I tried to use my board as much as I could. That was more interactive… more student participation…students were more engaged.
> If the lesson is going fine I pull up an enrichment activity which is online (Beth, personal communication, May, 2014).

She stated that the new textbook that the district purchased related to CCSSM, but that her school was using the state standards. She was still responsible for her students' fifth grade PASS scores. This made expectations confusing because, like Amy (in grade 4), they were expected to use the book addressing one set of standards, but teachers knew that they were ultimately still responsible for the state standards when it came time for end-of-year testing. She said in-service meetings were very basic and vague and discussed how teachers were using the new book. They were given a pacing guide but it also was vague. The district representative visited each grade level quarterly during their planning period to demonstrate an online option for an activity. Those meetings were brief and there was no time allotted for questions or extensions. She expressed that many teachers were resistant to the change because of the amount of time involved in preparing for each lesson and that her grade level primarily worked independently from each other. Each teacher wrote the plans for a subject and shared them with the others at their grade level. Beth was in charge of writing the math lessons for her grade level.

> We implemented a new book which was Common Core which no one knew and then we had an online thing. It was more kind of every man for himself because the school didn't actually implement the Common Core Standards like the other schools in the district. So we were using the State Standards. I was actually in charge of planning the math. So when I did it, I used the book as much as possible but then had to pull resources as needed for State Standards (Beth, personal communication, May, 2014).

Beth expressed frustration over a lack of training opportunities. When I asked why she felt that there is not more of a push for manipulatives in the school, Beth reported that the manipulatives had come with the new CCSSM series that they had ordered but that most of them had not been opened. She said the series did not explain how to use them and that there had been no training on how to use them. She expressed frustration with the lack of administrative support in terms of in-service training on the expectations for mathematics instruction and the use of the manipulative materials that accompanied the textbook.

> I don't understand why we haven't had more training with them. We had meetings for "My Math." That was the curriculum that we used. But she just wanted to talk about how we were using it (Beth, personal communication, May, 2014).

> We went to the "My Math" training but it was very basic and vague. We had one lady who came in a couple of times each semester during our planning and she basically asked us where we are and how things are going. Each time she met with us she would try to show us something online, but we didn't meet for very long. So again, it was very quick. If you got it you got it, and if you didn't you didn't (Beth, personal communication, May, 2014).

Time was an issue. Beth described there being a lack of time for preparing materials for lessons and a lack of time during the allotted math time to complete instruction due other activities, like computer lab, cutting into their math lesson. She felt that on the occasions when she did try to use manipulatives, students seemed to complete their work faster doing step-by-step problem solving using paper and pencil equations.

> I have all those manipulatives over there (points to two boxes) but I just haven't had time to look at them and see what's there and what to do with them (Beth, personal communication, October, 2013).

> Most of the teachers at the school are not willing to change, and that's understandable because it's more time preparing for it and you have to teach yourself (Beth, personal communication, October, 2013).

> It takes more time to teach with the manipulatives because you have to prepare for it (with the materials) and you have to teach yourself how to do it (Beth, personal communication, May, 2014).

> It took a long time. I think only a few of (the students) had used base ten in third or fourth grade, but they were in the same class (Beth, personal communication, May, 2014).

She said that because students were not familiar with using manipulatives in earlier grades they were confused when they got to her and were seeing them for the first time. This created some slight behavioral issues such as throwing the blocks or building things instead of working on the skill.

> If you don't start them out early using things, they don't know how to behave with them. When I brought my base ten blocks out they were thrown across the room and flicked and I said 'no, no, no, these are not flicking blocks…they are building blocks! (Beth, personal communication, May, 2014).

Although the team "planned together," collegiality was limited because of how the planning was structured in isolation. When I asked Beth to discuss her greatest challenge in teaching mathematics, Beth reported:

> Finding resources on my own… I didn't have a lot of help from other teachers because one who had been there wasn't teaching math… and the other because he had to do science and we had to plan our own thing (Beth, personal communication, May, 2014).

In the focus group when discussing the most important influence on how she taught in the classroom today, Beth reported feeling like her classroom mentor during her internship had the greatest impact. She said that "she showed me how to download a lot of different things." Similar to Amy's report relating to her math methods professor's notebook, Beth reported that the most helpful thing to her was an online notebook of activities shared by her mentor. This statement was inconsistent with the lessons that I observed with the exception of one lesson on volume. When I probed further she reported that she no longer had access to the notebook, and reports on the lack of available technology in her school were inconsistent with her statement about using the online notebook of activities.

Carly

In terms of her content knowledge in mathematics, Carly felt that she was about average. She self-reported that she did poorly in her core math classes at university but that she did well on her Praxis. She said that part of the problem for her was trying to figure out when to use which formula. In contrast to her previous math classes, math became much clearer when she took her methods class where they started using manipulatives for the first time. She also rated her mentor very high in the use of manipulatives. Carly began to understand mathematical concepts more when she started to teach math and saw how the concepts they learned in the early years fit into skills required later.

> I did awful in my classes. I think a lot had to do with the professors. I really struggled to be able to make a C in those classes. I had a tutor during those semesters. There was a lot of algebra and geometry and a lot of memorizing and formulas. Part of the problem for me was trying to figure out when to use which formula (Carly, personal communication, October, 2013).

> I had Dr. Bob (pseudonym) for math methods. She talked to us like we were elementary students. Things clicked that I hadn't understood before. We used manipulatives for the first time, other than rulers and protractors and that sort of thing. This was the first exposure I had had with the Tools kit (Carly, personal communication, October, 2013).

Carly indicated that she has a thorough understanding of elapsed time, place value, geometry and measurement. She said that she had a "pretty good" understanding of algebraic concepts and probability, and felt weakest in fractions and decimals.

I probed with the question, "I think it's interesting because you said earlier about the core classes that there was a lot of algebra and geometry and that was difficult for you. Here you mention that you feel really good about your ability with algebraic concepts and geometry. What changed? What happened that made you feel more confident in these areas?" She responded:

> I started to understand it when I got to teach it and saw how things worked…how things they did in second grade fit into how things worked later. Collaboration with other teachers made a big difference in this when they would explain how they taught certain concepts, it made me understand it as a learner better (Carly, personal communication, October, 2013).

When I asked why she was not using base ten blocks when teaching word problems with digits in the tens and hundreds, she explained that in the word problems she felt that they were struggling more with the wording of the problems than the numbers. She felt that this year she did more modeling than she has in the past as this seemed to be a lower group. Then she added that CCSSM was asking more of them than the state standards.

> I do a lot of modeling the lesson, then they come up to the board while everyone else is still sitting on the carpet. I felt like this particular group needed a lot of modeling, as compared to my previous two years of teaching. These were lower babies than what I have had in the past, and they have responded well (Carly, personal communication, May, 2014).

CCSSM has changed her way of teaching. It encourages her to think of multiple ways to learn content. She and her team struggled last year with CCSSM because it was unfamiliar and much more rigorous. This year was better.

> Common Core has changed our way of teaching. The rigor is insane compared to what it was with the State Standards. Students know way more coming from first grade. On the flip side of that, how Common Core was implemented, some students will fall through the cracks. If they started out with one set of standards and then jump to Common Core in fifth grade, there is going to be a big gap. Another thing that has been helpful for me is having to know multiple ways to learn the same content (Carly, personal communication, May, 2014).
>
> I think my instruction was better this year than last year. Last year we started Common Core. So this year I think I had a better grip on it than I did last year just because it was a lot more rigorous and it called for them to do a lot more (Carly, personal communication, May, 2014).

The use of the textbook that was adopted by the district has been controversial. Carly's team (and other teams, as reported by Carly) feels that the book does not address the needs of the students or the teachers and that the district is not helpful in how the implementation of the textbook should appear:

> I don't feel like we should just go by the book. And I feel like administration's hands are tied. They can't do anything about it because they have to answer to the district (Carly, personal communication, May, 2014).

Carly expressed frustration in the lack of district support. She reported that there was some confusion about how to use the new series to implement CCSSM. She expressed that she and her colleagues did not receive answers to the questions they asked and no strategic workshops had been offered. Like Amy and Beth, Carly reported that their instructions from the district level for implementation of the new series and CCSSM were vague. Initially they were told to follow the book explicitly. Later they were told the book was simply a resource. She and her co-workers started creating their own lessons to address the skills.

> We do not have the support from our district. They implemented a new curriculum this year. We don't even use it…..it's a spiral method and it's horrible (Carly, personal communication, May, 2014).

> (The coaches) are no help to us. When we ask them how to help us go further than this, they tell us to just go by the book (Carly, personal communication, May, 2014).

> We have had our coaches come out here, but we are not really getting the feedback that we need." The leadership team is from each grade level. They represent your team. You take the concerns to them and they take it to the higher level (Carly, personal communication, May, 2014).

> Originally we were told by the district coaches to use this textbook in its entirety…back to front. Later, they took back what they said,

> so now we're supposed to use it as a resource (Carly, personal communication, May, 2014).
>
> We are told that we can go to our math coaches and tell them things like what we need…I've never done it. I know other teachers that have done it don't get anything out of it. A lot of times they say, "here's an app for that" or "here are some tools you can use - figure it out" kind of thing. It's missing the explanation component (Carly, personal communication, May, 2014).

Like Amy (who teaches in the same district), Carly reports that there is a big technology initiative in her district. She uses MAP testing to assess skills in mathematics. She also uses IXL for skills practice and assessment. IPads and classroom computers are used regularly for mathematics instruction. Promethean boards are found in every classroom. There is a technology person who has met with Carly's team during planning and presented an app each time, but the team did not choose math. The team shares mathematics apps with each other when they find them.

> We are pushed to use technology in general because we are part of the GoTec (pseudonym) initiative, so we are pushed to use technology in general. We all share things. There are some great time and base ten blocks apps when we teach place value and it groups them into tens and connects them (Carly, personal communication, October, 2013).
>
> I have four iPads here in the classroom. If we find that a child does not have access to a computer at home we provide them an opportunity to work on a computer in class (Carly, personal communication, October, 2013).
>
> I used IXL that has all of the Common Core Standards for math, and I am able to go back and assign them a specific thing to do, and I can go back and see how they did and get a grade that way (Carly, personal communication, May, 2014).

Carly believed that time constraints was another big issue for not using manipulatives in her mathematics instruction. She reports having all the materials that she needs, but that sometimes it is faster to just write about what they learned or draw a picture of what they did. She reported wanting to teach in the way she had been taught in university but that “there’s not enough time in the day.” She said she now feels like she can get the same outcome finding other ways that are still fun and engaging without using manipulatives.

> I think that too often I choose not to use the manipulatives because of time. I guess I feel like it can go faster sometimes with just writing about it, say it out, drawing it. I can get the same outcome within less time (Carly, personal communication, May, 2014).

> I think that Oxton led me to believe that I would have way more time to teach meaningful lessons in a really fun way, and that time is just not there (Carly, personal communication, May, 2014).

> I still make a point to use a couple of those things that we did, like making the ladybug clocks that we made in Dr. Bob’s class. That is something I still take the time to do because it’s something they can take home and they still get to practice with it. I don’t feel like I’ve wasted my time, an entire math lesson making something cute that they’re not going to use later on. I still see students that pull their ladybug clocks out…just picking and choosing (Carly, personal communication, May, 2014).

Collaboration with other teachers was very helpful when Carly and her grade level shared their teaching strategies. Carly’s team planned and worked closely together. They discussed multiple ways of teaching concepts and tried to plan activities where students were moving and actively engaged. Carly reported that collegiality had the greatest impact on her teaching practices. She mentioned her team, her colleagues, her co-workers, and other teachers consistently throughout both interviews and the focus group. She felt that having a couple of years of experience working with this group of colleagues made a significant impact:

> Well, we plan as a team. We take the Standard that we do first and create an assessment then based on the assessment decide what our lesson is going to be. We all have the freedom to change the lesson as we want but we usually talk about multiple ways to teach it in

> hopes that one of those ways will click with each child. And the teachers are very good about sharing anything you need (Carly, personal communication, May, 2014).

> I think experience is the biggest thing. College made me think about cutesy all the time. All of the activities had some kind of neat cut-out or something to go with it. They didn't mention that we would have time to do a lot of these things. Collaboration with other teachers has made a big impact on me. Sharing ideas and learning from them and working together (Carly, personal communication, May, 2014).

Furthermore, Carly discussed the benefits of not only planning with her grade level team, but reaching out to other teachers in the school who may have different ideas for teaching mathematics. This was evident when she said,

> I get more out of going to other teachers and saying "what did you do?" Even teachers that aren't on my grade level that may have a different perspective as our PLC (professional learning communities) are good resources. We just have a lot of collaboration! (Carly, personal communication, May, 2014).

This statement was supported in her discussion of the team planning a geometry lesson with the use of 3-D shapes. All of the teachers shared the set they had with each other when it was taught so that each class had enough to use during instruction.

Comparison of Instructional Methods and Perceived Influences

Table 4.5 displays the instructions methods and participants' perceived influences of those practices.

Table 4.5 Instruction method and perceived influences

Instruction Method and Perceived Influences							
			Instruction method				Perceived Influences
			Manip.	Pic.	Wri.	Ver.	
School	Amy	Count	6_a	28_a	23_a	52_a	-trying to learn CCSSM
		% w/in School	5.5%	25.7%	21.1%	47.7%	-district emphasis on the textbooks
		% w/in Inst. method	15.8%	37.8%	40.4%	37.7%	-isolated from peers during planning -much communication via e-mail
		% of Total	2.0%	9.1%	7.5%	16.9%	-concerns with student behavior -district in-service was not helpful
	Beth	Count	12_a	27_a	17_a	47_a	-trying to learn CCSSM
		% w/in School	11.7%	26.2%	16.5%	45.6%	-district emphasis on the textbooks
		% w/in Inst. method	31.6%	36.5%	29.8%	34.1%	-isolated from peers during planning -minimal concern with behavior
		% of Total	3.9%	8.8%	5.5%	15.3%	-district in-service was not helpful
	Carly	Count	20_a	19_b	$17_{a, b}$	39_b	-district emphasis on the textbooks
		% w/in School	21.1%	20.0%	17.9%	41.1%	-reported strong collegial support
		% w/in Inst. method	52.6%	25.7%	29.8%	28.3%	-student behavior was not an issue -district in-service was not helpful
		% of Total	6.5%	6.2%	5.5%	12.7%	

All three participants were working with a newly adopted textbook. Additionally, this was the first year for Amy and Beth to work with CCSSM, therefore much of the time that they had for planning was spent becoming familiar with the standards. Concerns about learning the accompanying textbook and the district expectations took away from time that they felt they could spend trying to plan activities that incorporated manipulatives effectively.

Carly, who used manipulatives most frequently, had a strong support group at her school. Her grade level planned together weekly, touched based daily, and shared materials. Beth, who used manipulatives in the middle range, planned the mathematics in isolation and gave the grade level the plans each week. Amy, who used manipulatives least, often communicated with the teachers via e-mail. For Carly, group planning helped provide opportunities for multiple approaches to teaching mathematical concepts. Teachers "bounced" ideas off one another to plan for a variety of strategies in order to differentiate instruction to meet the needs of their students resulting in more activities that used concrete and pictorial representation. Amy and Beth, who did not have strong group support, relied more heavily on the textbook, which used more written representation. For them, verbal representation was utilized for explanation of textbook activities.

Carly, who used manipulatives most frequently, felt that behavioral issues were not a problem in her classroom. The perception of a lack of behavioral problems allowed her to try more activities that encouraged student movement and active engagement. Beth, who used manipulatives in the middle range, reported some behavioral issues but that it was not a huge deterrent from using manipulatives. The perception of minimal off-task behavior did encourage her to use manipulatives less often and use technology and textbook activities instead. Amy, who used manipulatives least, reported that behavioral issues were an on-going problem and a factor in her choice to use technology or paper/pencil activities in lieu of manipulatives.

Focus Group

At the end of the school year in May (2014), all participants met for a focus group discussion about their instructional practices in mathematics, particularly their use of concrete representations, and the perceived influences on the instructional decisions they make. I asked participants to share some of the ways that they used concrete representations (i.e. mathematics manipulatives) in their classroom instruction this past year. The open style of the focus group afforded participants to discuss aspects of instructional decision-making they felt was relevant to them. Participants discussed a variety of manipulatives that they used and some of the ways that those manipulatives were used during their instruction. The following exchange is indicative:

> BETH: We used nets most recently and the kids really got it when we took it from the flat paper and built it and I had some built and they had to deconstruct them.
>
> CARLY: We did the same type thing. We did the same thing with marshmallows and toothpicks making 3-D shapes. We also did money and clocks.
>
> AMY: We did manipulatives mostly with measurement. We used our iPads for fractions. I consider an iPad a manipulative. So we used those a lot for fractions and decimals…and then assessment.
>
> BETH: We did the fractions bars with paper.
>
> CARLY: We used candy for making graphs, and then I also used the yard sticks, rulers and tape measures. We used items around the room for benchmarks.

Within this discussion, teachers began commenting on nuanced differences in the Common Core State Standards for Mathematics and their adopted textbooks. This conversation started with a discussion of different meanings of benchmarks, then transitioned to a fidelity to adopted textbooks.

> RESEARCHER: Your definitions are different. Was this a district vocabulary word? Where did the term "benchmark" come from?
>
> AMY: It's in our Common Core Investigations.
>
> BETH: We didn't do Common Core math at my school.
>
> CARLY: Did you all use Investigations in its entirety or as a resource.
>
> AMY: Bits and pieces.

CARLY: We used it as a resource but our book didn't really tell us what a benchmark was.

AMY: No, ours didn't either.

CARLY: That was in our Common Core standards, but I couldn't find it in any of the lessons …that terminology being used.

AMY: I had to read and kind of figure out what that meant because I didn't know what it was either.

AMY: Investigations is our newly adopted math series. They might change it to try to meet Common Core.

CARLY: I hope so. I thought that was why it was chosen because it was supposed to match Common Core.

Although working with the CCSSM has presented some challenges for the districts, and consequently for the teachers and the parents, overall, all three participants were happy with the rigor required by the standards and felt that it would be a positive thing once the district had worked out some of the kinks and teachers had learned effective implementation and efficient assessment strategies.

CARLY: My team has enjoyed the Common Core with math because it has pushed our children a lot further. When I compare them to last year, my kids have done amazing things that we didn't think they were capable of. We liked it for math.

AMY: I agree. I like it for math.

CARLY: I like the rigor that is required.

BETH: And I like it too, but I feel some of the stuff I had to do with my kids, they weren't used to it yet. It was very difficult and so it put them down even farther because they thought "I can't get this."

AMY: Mine are like that too.

CARLY: I think with Common Core, too, number sense is an important thing. It requires them to have number sense and there's a gap. But just to see the difference in the second graders this year compared to the ones I had last year that had Common Core last year. Now that I have them, they have so much more number sense just from that one year of Common Core.

BETH: But you need to start off early because my fifth graders are lost.

CARLY: And we need to address the huge gaps.

The discussion returned to the use of textbooks and other curricular resources made available by the district. Teachers stated that the order of mathematical concepts were problematic; the gaps that were discovered made effective instruction difficult.

AMY: We had a curriculum map but there were HUGE gaps.

CARLY: I agree. The spiral thing is frustrating to me. I know that it's a good concept that you are constantly reviewing, but I feel like there are too many gaps between what they are spiraling in. You are non-stop review all the time.

AMY: And the gaps…We looked at the 9-weeks parent sheet that our district makes, and we did the first book of Investigations. We got to the end of the nine weeks where we had to put it in the sheet and half of the Standards weren't even in the book.

BETH: We adopted a curriculum map from another county, but it wasn't a district-wide adoption.

CARLY: I think that's one positive thing about Common Core. In our district if I have a child that goes to another school they follow the same thing, so they will pick up right where they left off at my school.

CARLY: Exactly…which we are guilty of that because, like I said, they didn't want us to teach place value until after we taught addition and subtraction and that didn't make sense. So we didn't follow that rule necessarily…But for the most part, we tried to stick with it.

When I asked about some of the things that have been helpful in encouraging the use of manipulatives in their math instruction, the conversation quickly shifted to some of the obstacles relating to district support. In all three cases, the district meetings were brief (limited usually to within a planning period allowing about 20-30 minutes), seldom (averaging a minimum of once a quarter), did not address all of the participants (inviting two representatives from the school), and did not provide opportunities for hands-on practice for the teachers. Further, no plans were made for how the information collected by the representatives would be distributed to the remaining faculty, often relying on e-mails as opposed to face-to-face reports of information.

> AMY: I was on Team Math, which is district-wide, two representatives from each school…and they were trying to help us out with the Investigations. Most of us just don't like (the book).
>
> CARLY: I don't know who our "Team Math" representatives are.
>
> AMY: There were one or two representatives from each school, but we were supposed to go back and we could tell our teachers. We went to the meetings and if we thought it was useful we could tell them. So we would send them an e-mail and then if they had questions they could come to us...but they didn't really ask.
>
> RESEARCHER: So it was a small scale initiative?
>
> AMY: Yes.
>
> BETH: We had a representative that would come every other month or so, and each time she would pick out something in the book. We also had an online thing so I could pull the virtual fraction bars online but half the time my Promethean board didn't work with my tablet … so each time we got about 20 minutes of all this information but no practice and she was always traveling, so it was frustrating. We kind of had to teach ourselves what we would do. So we did have some help but it wasn't very specific.

> AMY: Team Math was like that too because we did activities for kindergarten, which is great for the first grade representative, but it had nothing to do with what we were doing.

> AMY: There's some website that goes with Investigations that they told us how to sign up for that, but that website was really confusing so…none of us signed up for that.

> CARLY: My team signed up on the website but we had a million codes. You had a code for yourself, one for the team….and that was super confusing but, once we got into it, it had the equivalent of flip charts but they were designed through the actual Investigations program. Some of those were good but you had to weed through a million to find the five that were really useful. It was kind of like working harder to find something that would go with your lesson.

When I asked about some of the roadblocks that prevented them from using manipulatives in their instruction, a variety of concerns emerged. The first was frustration with the lack of professional development from the district to support the use of the new materials, followed by the limited amount of time that teachers had for preparation and implementation.

> BETH: Lack of training. I was given two big boxes of stuff and both boxes had some of the same stuff so it was all mixed and mingled and in the book it didn't say use this with this and so some things were really random.

> BETH: It was a whole different thing using "My Math" but doing state standards because, yes they are correlated somewhat, but I had to go online and find resources and things all on my own a lot which was rough. I was given the boxes. We had the 20 minutes training. But that was only online stuff. None of us knew "My Math." It was a big struggle this year.

> AMY: Investigations was kind of like that, too. For mine it gave us games to play and a big number line to make when we did fractions. And there was all this stuff to do but it took four hours to get it all ready. I have a ton of stuff but unless we did it at Oxton, I didn't really know what to use it for.

As was apparent during the initial interviews, student learning levels and behavior arose during the focus group. This seemed to be more of an issue for Amy than for the other two participants, but Beth shared that she occasionally has to deal with difficulties in participation.

> AMY: My kids can't handle it. Even if you give them ten minutes to play with the shapes, they're still building something 20 minutes later. They just don't know how to talk to each other kindly. They're just not nice.
>
> BETH: Mine is kind of the same way. Part is lack of participation. They don't have the confidence to try.
>
> CARLY: Mine have no problem getting along. But I think part of that is age. They're second graders. In second grade they all love each other for the most part.

I asked participants to talk a little about their teams because I noticed a distinct difference between all three participants. I did not hear Amy mention her grade level team at all, while Beth mentioned meeting with her team for the purpose of exchanging the written plans as opposed to a collaboration effort. Carly, by sharp contrast mentioned her team consistently in both interviews and during the focus group.

> BETH: I had a great team, it's just we all did our own ELA planning. Mr. James (pseudonym) did science, I did math, and Ms. Flannigan (pseudonym) did Social Studies.
>
> RESEARCHER: So the sharing of ideas was not really happening so much?
>
> BETH: Our planning was done on an online planner so you can copy and paste it, so whatever I did for math, I had to have it online ready for them to copy and paste by Friday at 3:00. But sometimes the materials weren't with the subjects. So I have a great team but our collaboration isn't strong.

CARLY: Well so what happens when you don't like what that team member came up with? Do you change it or do you use it?

BETH: I change it. I was teaching the same things that she was teaching but just teaching it in a different way.

CARLY: I think that I would be concerned about not having ownership if I just copied and pasted what they came up with and then got in front of the room to teach it. That's what I really like about our team because we sit down and plan every subject together and everybody throws their ideas.

RESEARCHER: How much time is involved in that?

CARLY: About an hour a week…we're basically planning for two subjects during that hour.

BETH: We got together, said what our plans were. That's when we could ask questions or make suggestions. And then that was it. There was not much more discussion.

AMY: We meet on Wednesdays for planning and we kind of look at the plans from last year. We met once a week.

AMY: I can go in to my mentor with any question that I have and she will help me. She will help me if I ask her, but it's kind of like both of us trying to figure it out together since she was in fifth grade last year.

When asked what participants would say was the single thing that impacted their mathematics instruction the most, the responses were mixed with Amy feeling like it was her university coursework, Beth adding to her coursework her mentor, and Carly stating her mentor and the team with which she teaches.

AMY: I'd say for me it was Oxton more than my mentor and more than my district. Dr. Bob…I've gone back in her notebook that we had to buy and have gone back and looked at her stuff this year. So I'd say Oxton for me. I'd say I learned more from her than anybody else.

BETH: And I'll agree and I'll also add my mentor. We did more things…student led, student made Investigations in math, versus using a book, that I've been able to incorporate this year as I didn't have much knowledge of it before.

AMY: Dr. Bob gave us a lot of activities that we did. For me it was like when I needed something to get them interested, I could go look in the notebook that she did for us. It was more of resource for me than the district.

BETH: (My mentor) went on "Teacher Pay Teacher" and downloaded this, math notebook. It was a daily binder of different activities you could do in a book in order all together.

RESEARCHER: Do you use that book this year?

BETH: I don't have it. I need to get in touch with her and get that link.

AMY: Like an interactive notebook?

BETH: Yes.

CARLY: I would say mine is more my mentor and my team. I had great team support and I still do. We just have a lot of collaboration. I feel like Oxton taught me a lot but I feel that Oxton made me think that that I would have a lot more time. I guess Oxton gave you just a different outlook on what teaching is than when you actually got in the classroom.

CARLY: I think that support from your team members is the best thing, even above the mentor. I mean I had a great mentor. I couldn't have asked for anyone better. But I got put on a team that everybody works well together. Everybody wants to share. There's great collaborations, so you're getting five minds put together to come up with the same outcome.

Carly's final two statements in the focus group were consistent with my observations of her lessons, the discussions in our interview sessions, and throughout the focus group discussion. Beth and Amy both reported that their university professor (and Beth added the mentor) had the most influence on their current teaching practices. These statements were inconsistent with my observations and discussions, which found that both participants relied heavily on the textbook for instructional guidance. .

Discussion of the Relationship between Concrete Representation Usage and Influences as Perceived by Beginning Teachers

All three participants had ample access to manipulatives, yet they used them to significantly different degrees. Although they shared similar pre-service experiences in relation to their coursework and internship, there were substantive differences between them in relation to how they view Common Core (CCSSM), how they interact with their colleagues, how they perceive their students, and how they perceived district involvement. This seems to result in different implementations of concrete representations.

CCSSM was a subject of concern for all three. As first year teachers, both Amy and Beth expressed concerns about learning and teaching to the standards. Carly, in contrast as a third year teacher, felt more comfortable with the CCSSM standards, having worked with them the previous year. Carly was the first to endorse the implementation of CCSSM, and Beth and Amy did agree that CCSSM was good for the students as long as how those standards were to be addressed were clarified to the teachers. All three participants felt that how the standards would be implemented within their districts was not clarified by district personnel. All three were in districts who had newly adopted a text meant to address CCSSM. Beth relied heavily on the textbook as a guide to address the standards, whereas Amy used the textbook from the previous year. Carly used the textbook as one resource, but planned with her team to find outside sources in order to supplement and enrich learning.

Carly had a strong support group at her school as she worked with her team members, other teachers within the school, and the administration. I noted that the two least experienced teachers in the study were both in leadership roles in relation to mathematics. Amy was the "Team Math" representative for her school, and Beth was in charge of writing the weekly lesson plans in mathematics for her grade level. Neither had extraordinary support from colleagues unless they specifically inquired, and both frequently made instructional decisions independently.

Due to perceived behavioral problems in the classroom when manipulatives are used, Amy opted for the use of technology or pencil/paper activities in lieu of concrete representations in order to manage the classroom. Although Beth reported similar issues with behavior management, it was only a slight deterrent from using manipulatives. Beth also frequently opted for technology in lieu of manipulatives. A greater consideration for Beth in her instructional choices was her high comfort level with technology versus the amount of time needed to become familiar with the manipulatives and how they should be used for specific skills. Carly was also concerned about the time issue, but her concern related more to implementation of the lessons and student completion of tasks while using manipulatives. She did not have classroom management issues that she perceived could be a deterrent from using them.

All three participants expressed frustration in the lack of guidance from district personnel. (Although Amy expressed some concern, she was the least phased by the lack of district involvement). Cursory in-service meetings were not perceived to be helpful. Meetings were brief and seldom, leaving participants to feel unsure of expectations for their instruction. Presentation of new material left little time for questions, clarification, exploration, or practice. This disconnect between teachers and district left teachers to pursue individual strategies for mathematics instruction.

Conclusion

Participants made use of concrete representations for teaching mathematics by using paper clips and rulers for linear measurement, nets and 3-D shapes for investigating three-dimensional shapes, base ten blocks for place value, coins when exploring money, clocks for exploring time, and candy for making graphs. These uses of manipulatives that were observed during the lesson and discussed during the interviews and focus group represented only a small portion of what participants reported to be part of their university methods course or their pre-service clinical experience. In addition to concrete materials, participants also utilized technology for representing Geoboards, fraction pieces, and virtual rulers. IPads were utilized at all three schools, but availability to students varied significantly between schools. They also implemented the use of technology for posting charts and tables, as well as for assessment. Manipulatives were present and accessible at all three schools; however, they were used least of all five of Lesh's (1987) representations.

According to all three participants, prior experiences impacted their use of concrete representations by exposing them to the use of manipulatives in their math methods class and, to varying degrees, in their internships. All three participants expressed different levels of comfort with their knowledge in mathematics when they were in school and at the university. They unanimously expressed having exposure to math manipulatives in their pre-service experiences, but differed in the amount of exposure they felt they had. However, through this year-long study, participants communicated that their decisions were based more around their current teaching contexts than teacher education experiences in relation to whether and how teachers used concrete representations for mathematics instruction than did their pre-service experiences. Emerging themes were technology, behavioral issues, CCSSM, textbook-related matters, time constraints, professional development, and collegial collaboration.

Current teaching contexts significantly impacted beginning teachers' use of concrete representations. Two participants within the same district said that specific terms were not clearly defined, therefore their interpretations and their instruction relating to those interpretations were very different. Both districts adopted textbooks that were marketed to address the CCSSM standards. All three participants conveyed frustration regarding the vagueness in relation to addressing CCSSM.

For all three participants, they were strongly encouraged to use the textbook as their curriculum. None of the participants were particularly pleased with their textbook series, but all three used it in varying degrees. All three reported having to create some of their own lessons in order to address concepts that were omitted from their current textbook. Participants perceived a lack of guidance from their districts, citing mixed messages relating to the implementation of their mathematics instruction, lack of professional development, and minimal face-to-face contact time. Participants conveyed a lack of explanation and follow-up relating to professional development.

All participants agreed that time was an issue. These issues included searching for different resources, preparing materials for lessons, and time to actually finish a lesson without interruption. For two participants, behavior influenced the use of manipulatives for math instruction, one significantly. For these same two participants there was a lack of interactive team support in their mathematics planning. Although two of the participants indicated that they felt that their pre-service experiences primarily influenced their teaching practices, there was little evidence to support these statements. In the case of all three participants, based on the observed lesson plans, individual interview discussions, and the focus group discussion, primary influences were related to the context of their current teaching environment.

Researcher's Observation

Technology was used at all three schools, but availability ranged from seldom for individual students to every student having daily access. There is a possible representation that was not initially considered for the purpose of this study. Technology appeared in many forms during this study: Promethean boards, Edmodo, iPads, instructional websites, purchased programs, various apps, etc. Figure 4.1 suggests a possible model for consideration. Within this model, I have added the representation of technology, but I have also added some of the possible influences relating not only to pre-service experiences but also incorporating possible influences within their current teaching contexts.

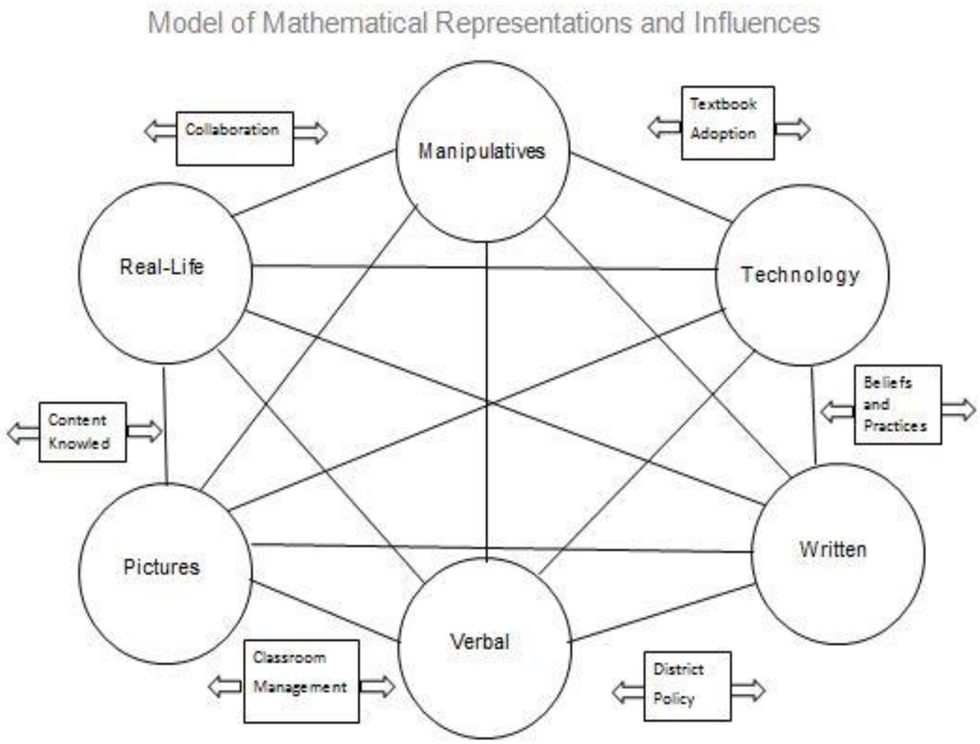

Figure 4.1 Model of mathematical representations and influences

This model depicts in the circles the six types of mathematical representations: manipulative, technological, pictorial, verbal, written, and real-life. All of these representations are interconnected, and teachers and learners can move between and among the representations in a non-linear path. The boxes on the outside of the representations represent the things that influence the use of representations at any given time. Those things include issues within the teaching context such as district policy, beliefs and practices, textbook adoption, collaboration among peers, content knowledge, and classroom management. The arrows emerging from the boxes show a movement of influences. At any given time some things may have more of an impact than others. For example, if district policy mandates the adoption of new standards or a new textbook, those influences may be more closely related to the instructional practices and use of representation than others. If there are years that a teacher perceives classroom management to be problematic, she may rely on more written and technological representations in or to maintain structure within the classroom. In other words, at different times, any of the outlying influences may have an impact on whether and how frequently various representations are utilized. Implications of these findings will be discussed in Chapter 5.

Chapter V

DISCUSSIONS, IMPLICATIONS, AND CONCLUSIONS

This mixed-methods case study documents the pre-service experiences and teaching practices of three early childhood/elementary grades mathematics teachers. The relationship between pre-service teaching experiences and teaching practices was explored by asking two questions: (a) how do beginning teachers make use of concrete representations for teaching mathematics and (b) in what ways prior experiences and current teaching contexts impact beginning teachers' use of concrete representations? Focusing on the use of concrete representations for teaching mathematics, this study examines a collection of quantitative and qualitative data, including video-recorded observations with accompanying artifacts, interviews, and a focus group.

Overview of the Study

Theoretical foundations for this study were grounded within the broad idea of constructivism (Brooks & Brooks, 1993; Condon et al., 1993; Culatta, 2012; Fosnot, 1996; Hsueh, 2005; Khalid & Azeem, 2012) and Lesh's (1987) analysis of modes of representation with an emphasis on the use of concrete representations (manipulatives) within this model. Because of the abstract nature of mathematics, its ideas are often communicated through different types of representations, which are symbolic depictions of mathematical thinking. The opportunity to use representations is vital to the process of building mathematical understanding for students (Fennell and Rowan, 2001; Goldin and Shteingold, 2001; Lamon, 2001; Moyer, 2001; Lesh et al., 2003). Mathematical thinking can be “represented” through pictures, written word, or verbal communication, manipulatives and symbols (Lesh, 2003; Goldin & Shteingold (2001); Kamii, Kirkland, & Lewis, 2001; Sowell, 1989). Research from both theory and classroom studies have shown that the use of manipulatives for instruction in mathematics positively affects student learning (Burns, 2005; Cass et al., 2003; Cramer et al., 2002; Grouws and Cebulla, 2000; Kelly, 2006; McCarthy, 1987; Munger, 2007; Olkun & Toluk, 2004; Reys, et al., 2007; Ruzic and O’Connell, 2001; Sowell, 1989; Sutton and Krueger, 2002). The National Council of Teachers of Mathematics Standards (2000, 2006) and Common Core State Standards for Mathematics (2010) both stipulate that students need opportunities to solve problems and have multiple occasions to communicate mathematical ideas with others. Representation using manipulatives is essential in teaching mathematics at the early childhood/elementary school level, prompting the Nation Council for Teachers of Mathematics (NCTM) to recommend in their Principles and Standards for School Mathematics (PSSM) that students be able to 1) use representations to organize, record, and communicate mathematical ideas, 2) use and move among mathematical representations for problem solving, and 3) use representations to model and interpret mathematical occurrences (PSSM, 2000.).

The National Mathematics Advisory Panel (2008) recommends that research should look at effective instructional practices and ways to enhance teachers’ effectiveness. In this study a mixed methods approach was used to investigate how new teachers within the first three years of teaching in their own classroom, used manipulatives for mathematics and in what ways prior experiences and current teaching contexts impact their use of manipulatives. This chapter includes a summary of the study, implications of the results, recommendations for future research, and conclusions.

Data Analyses & Results

Three sources of data were collected: interviews, video-recorded lessons referred to in the study as "scoops" (Borko et al., 2007), and a focus group. An initial interview protocol was used to understand the attitudes toward mathematics and pre-service experiences of each of the participants. The use of multiple sources helped triangulate data ensuring that what was seen and heard occurred in multiple instances and from multiple sources.

Each of the three teachers who completed the study shared a similar university experience from a common university; however, each participant taught at a different school, taught a different grade level, and had a different extent of classroom experience. Each participant expressed her own views in terms of her pre-service experience and her current teaching context. While some views shared were quite similar, there were some distinctive differences in teaching practices and perceived influences of those practices. Teachers participated in two interviews and their lessons were video recorded over ten days of instruction. Two levels of analysis were conducted. The first was a statistical analysis of the frequency of the use of five different mathematical representations (Lesh, et al., 1987) during mathematics instruction, and the second was a thematic analysis of perceived influences of the use of those representations during mathematics instruction.

Summary of Findings for Question One: Use of Concrete Representations by Beginning Teachers for Teaching Mathematics

To address the first question, I video recorded beginning teachers' mathematics instruction for a two week period, coding the representation types as discussed by (Lesh et al, 1987), which were apparent during video clips of instruction. I ran a frequency distribution to determine the percentage of the use of each representation, and conducted interviews relating to their instructional decisions. Manipulatives were present and accessible, but they were used least of the five of Lesh's (1987) representations. Pictorial representations were used slightly more than concrete manipulatives whereas abstract written symbols were used more often than either of the former. This was an expected finding based on my observations of new teachers' practices in mathematics instruction over the last several years and based on discussions with the study participants during the interviews throughout the year. Verbal expression was most prominent and was used in isolation as well as in conjunction with other forms of representation. Real-life representations were not observed. Although technology was not considered as a factor prior to the study it seemed to be used consistently for math instruction. The methods and degrees of its use, however, varied significantly. These findings may suggest that, with the current capabilities of technology ever expanding, technology may be an effective component when considering various forms of representation in future research.

Amy

Among individual participants' manipulatives were used least often by Amy. She noted her regular use of manipulatives when introducing a topic, however this was inconsistent with findings from video recordings of classroom practice. Although Amy felt that she had a good understanding of how to use math manipulatives, she did not use manipulatives in her video-recorded instruction with the exception of rulers and protractors. The majority of representation use during her instruction was verbal. She used technological versions of manipulatives on occasion and utilized technology for skill practice and assessment. The majority of my observations of her instruction were formula and textbook-based. She frequently used the Promethean board to post charts from the text for the students to copy. She and her students used the iPads on a regular basis using the internet and various apps. As Amy discussed her use of manipulatives, she often discussed her use of a virtual manipulative or app rather than concrete materials.

Beth

Beth fell between Amy and Carly in her use of manipulatives. She stated that she had ample access to manipulatives that came with the series, but she did not use many of them in her instruction. When she did use manipulatives, she used rulers for linear measurement activities, nets that students folded into 3-D shapes, and snap cubes. Like Amy, Beth and her class also used verbal expression most often of the five representations. This statement was supported in the video recorded lessons. Also like Amy, she used the Promethium board consistently to display pages from the textbook and occasionally accompanying pictures. The majority of my observations of her instruction were textbook-led. Unlike Amy, she did not use any apps or iPads in the instruction that I observed.

Carly

Carly used manipulatives most frequently. She estimated her use of manipulatives for instruction to be more than half of her instructional time, which was more consistent with findings from video recordings of classroom practice than her counterparts. Noted manipulatives included fingers, rulers, 3-D shapes, and tape measures. Like Amy and Beth, she also relied heavily on verbal expression, but the use of manipulatives came in second. Carly rated herself above average in the use of manipulatives citing examples such as 3D shapes, counters, use of ladybug clocks, and measurement tools. She recounted that she did not use manipulatives much during the second semester.

There was significant variation in the *frequency* that manipulatives were used, but only a slight variation in the *types* of manipulatives that were used. All three specified using rulers for measurement, and their recordings were consistent with those statements. All three relied heavily on verbal representation. Amy and Beth used the Promethean board to display pages from the book, whereas the frequency of this practice was noted to be significantly less in the recordings of Carly's lessons. Of the three participants, Carly used manipulatives the most.

Summary of Findings for Question Two: Impact of Prior Experiences and Current Teaching Contexts on Beginning Teachers' Use of Concrete Representations

To address the second question, I conducted interviews and a focus group to discuss the instructional practices of each participant, the variation in instructional practices among participants, and each participant's perception of what influences her instructional practices. Using an inductive coding scheme, I put similar pieces together, assembled them into "chunks", and identified any linking pieces so that the big chunks could be connected (LeCompte, 2000). I coded the data in order to organize the information into categories so that the data could be organized and verified, the findings reported, and implications of the study discussed (Lincoln & Guba, 1985; Miles & Huberman, 1994).

Amy

Amy was confident in her knowledge of content for teaching mathematics. She indicated that she had had extensive exposure to math manipulatives in her methods class. She felt that she had ample access to multiple varieties of manipulatives and in her current teaching setting and felt she had a good understanding in the use of manipulatives. Impacting her decision to use the technology instead of the physical manipulatives were behavioral issues. Her belief that her students were not able to get along, share the materials, and stay on task when using the concrete materials influenced her decision to use technology options. Amy recounted several times that students could not work well together. Technology, especially the Promethium board and the iPad, was used frequently in her class to manage behavior. She frequently used a chart for students to copy into their journals.

Amy disclosed that this was the first year the school had implemented Common Core State Standards in Mathematics (CCSSM), therefore all of the teachers were still trying to learn it. She related that as her team studied the curriculum, they found that it was very vague and that there were several gaps in the curriculum. They relied heavily on the book for the first half of the year but began to use it more as a resource during the second half of the year. The unfamiliarity with the new CCSSM and beliefs that end-of-year testing would address standards from the previous year impacted her teaching practices as well as those of her colleagues. Teachers on her team were concerned that the state test did not address CCSSM, therefore the team felt that they needed to address both the state standards from the previous year as well as the current CCSSM. Amy recounted that there were no in-service training sessions on how to effectively use manipulatives in the classroom and that the district was pushing for more use of technology in the classroom, so most of the in-service trainings were focused on the use of technology and how to use the new textbook in order to address CCSSM. Amy described being a representative for her school on “Team Math,” which was compiled of two teacher representatives from each school and met quarterly. She did not convey that there was any support from administration or fellow teachers in this leadership role. Amy recounted in the focus group feeling that her university methods class and the prepared notebook of activities from that class had the greatest impact on her teaching practices and that it was much more helpful than the district meetings.

Beth

Beth felt that she had an average content knowledge base for teaching mathematics. Beth felt that in her elementary math methods class she had been exposed to a variety of manipulatives and instructional strategies using them. She felt that she had ample access to manipulatives, referring to boxes of manipulatives in the back of her classroom that were seldom used because the students weren’t accustomed to using them, there had been no training in how they should be used, and because she did not have time to go through them. Beth reported believing that the district wanted them to use the textbook as much as possible. Therefore, she relied heavily on the textbook and ultimately followed the curriculum map and plugged in where it could be found in the textbook. She and the other teachers used the book as their primary source and made adjustments in instruction as needed.

Beth described using technology whenever possible, but that her school was limited in the number of computers that were available to students. Internet access was intermittent which also created frustration. When possible, she used the online component of the series, posting pages from the text onto the Promethean board. Beth stated that the new textbook that the district purchased related to CCSSM, but that her school was using the state standards and she, like Amy, was concerned that she was still responsible for her students' PASS scores. Beth expressed frustration over a lack of training opportunities, stating that in-service trainings were brief, basic, and vague and focused on how teachers were using the new book. She expressed frustration with the lack of administrative support in terms of in-service training on the expectations for mathematics instruction and how to use the manipulative materials that came with the textbook series in order to enhance the activities in textbook. She noted that many teachers were concerned with preparation time and worked independently from each other. Beth described there being a lack of time for preparing materials for lessons and a lack of time during the allotted math time to complete instruction due other activities. She felt that on the occasions when she did try to use manipulatives, students seemed to complete their work faster doing step-by-step problem solving using paper and pencil equations. This statement speaks to her beliefs that being able to write the correct response embodies mathematical proficiency. She reported that using manipulatives sometimes created slight behavioral issues. Beth was in charge of writing the math lessons for her grade level and did not note having support from co-workers or administration in this regard. Collegiality was limited, as teachers planned for the group independently in each subject. Beth recalled that her classroom mentor during her internship had the greatest impact on her current teaching practices, although most of her concerns relating to why she did not use manipulatives more frequently related to her current teaching setting. Similar to Amy, Beth said that the most helpful thing to her was an online notebook of activities shared by her mentor.

Carly

Carly stated her content knowledge in mathematics was average. She struggled in her core mathematics classes at university noting that part of the problem for her was trying to figure out when to use which formula. She recalled that math became much clearer when she took her methods class where they started using manipulatives for the first time and rated her mentor very high in the use of manipulatives. Carly began to understand mathematics more when she began to teach it. Carly's experiences and use of concrete representations aligned with research that demonstrates an inverse relationship between content knowledge and reform-based teaching. That is, teachers with weaker mathematics content knowledge appear more apt to use reform-based instruction in their classroom (Wilkins, 2008; Ernest, 1989).

Carly related believing that the adoption of the CCSSM encouraged her to think of multiple ways to teach content. She said that she and her team struggled last year with CCSSM because it was unfamiliar and much more rigorous than previous standards but that they had a better understanding of the expectations going into the second year. She conveyed the use of the textbook that was adopted by the district had been controversial, citing that the book did not address the needs of the students or the teachers and that the district was not helpful in the implementation of the textbook. Carly expressed frustration in the lack of district support. She recounted there was some confusion about how to use the new series to implement CCSSM. Like Amy and Beth, Carly recalled that their instructions from the district level for implementation of the new series and CCSSM were vague. Like Amy, Carly related that there was a big technology initiative in her district, so much of the in-service was related to the use of technology in the classroom.

Carly stated that time constraints was another big issue for not using manipulatives in her mathematics instruction. She said that she believed that sometimes it is faster to just write about it or draw it. She disclosed wanting to teach in the way she had been taught in university but that there's not enough time. Collaboration with other teachers was very high, with the team planning and working closely together. Unlike Amy and Beth, who related their university experience and activity notebook to be the greatest influence on their teaching practices, Carly accounted that collegiality had the greatest impact on her current teaching practices in mathematics.

Relationship of Findings in Questions One and Two

All three participants had ample access to manipulatives, yet they used them to significantly different degrees. Although they shared similar pre-service experiences in relation to their coursework and internship, there were substantive differences between them in relation to how they viewed CCSSM, how they interacted with their colleagues, how they perceived their students, and how they perceived district involvement. CCSSM was a subject of concern for all three. Carly, however, felt more comfortable with the CCSSM standards, having worked with them the previous year. All three participants believed that the implementation of the CCSSM within their districts was not clarified by district personnel. Both Amy and Beth relied heavily on the textbook as a guide to address the standards. Carly used the textbook as one resource but, unlike Amy and Beth, had strong team support and planned with her team to find outside sources in order to supplement and enrich learning. Both Amy and Beth reported behavior management as being a deterrent from using manipulatives, but to differing degrees. Amy and Beth frequently opted for technology in lieu of manipulatives. Beth and Carly both said they were concerned about the amount of time it took to prepare quality lessons using manipulatives and then the time it took for students to use manipulatives within the allotted class period time. All three participants expressed frustration in the lack of guidance from district personnel. In-service meetings were perceived to be not helpful. Meetings were brief and seldom, leaving participants to feel unsure of expectations for their instruction. Carly's belief that she had the support of her colleagues empowered Carly, whereas Amy and Beth exhibited a more external locus of control.

Implications

After analysis of interviews with each of the participants and the discussion within the focus group, several themes appeared. Each of the participants in this study shared a similar pre-service experience, but their use of instructional approaches varied between them. What this study provides is an observation of the differences in how teachers used various instructional approaches and what impacted those practices.

Technology

Technology is becoming more and more visible in classrooms. Amy and Carly both taught in a district that had a strong push for technology, and Amy had access to a very strong technology coordinator who was quickly responsive and brings materials as requested. Virtual manipulatives may be considered a new form of representation or a distinctive combination of several representations (Goldin, 2003; Goldin & Shteingold, 2001). Virtual manipulatives, which are interactive, computer-based visual representations of mathematics manipulatives (Dorward, 2002; Goldin, 2003; Moyer et al., 2002; Moyer et al., 2008), were used by all three participants at some level. Although virtual manipulatives have some similarities to concrete manipulatives, they go beyond the capabilities of concrete manipulatives in that many have links between and among pictorial representation and symbolic notations. Virtual manipulatives have some of the advantages of several different representations, as well as some similar advantages of their technological properties.

Virtual manipulatives have another advantage. Virtual manipulatives are powerful tools in that they constrain the learner's action and direct the learner to focus on the mathematics in the virtual environment (Zbiek et al., 2007). Two participants specified student behavior as being an issue when trying to use manipulatives for math instruction. They recalled problems such as using materials inappropriately, arguing over the manipulatives, disrespect to peers when working with manipulatives, lack of participation, and lengthiness of lessons due to dealing with what they perceive to be off-task behavior. Amy stated that she chose to use the iPads with the class as opposed to having students share manipulatives in order to help with behavioral issues. Research has indicated that instruction using virtual manipulatives either alone or in conjunction with concrete manipulatives promotes gains in mathematics achievement (Bolyard, 2006; Moyer et al., 2005; Suh et al., 2005) and encourages student engagement and on-task behavior (Drickey, 2000). Findings from Amy's instruction coupled with this research are a powerful motivator for school districts to push for further use of technology in mathematics.

Teacher Understanding of CCSSM

CCSSM issues were consistently referred to during discussions. There seemed to be a lack of clarity in terms of some of the vocabulary used by the textbook that was adopted to address CCSSM. Two participants within the same district said that specific terms were not clearly defined; therefore, their interpretations and their instruction relating to those interpretations were very different. Although one district had adopted a series geared toward CCSSM, one participant's school had not incorporated the use of CCSSM standards for their mathematics instruction. This created some confusion among participants in where to place emphasis on certain skills. All three participants conveyed frustration regarding the lack of clarity regarding CCSSM. These statements were consistent with Cogan et al. (2013) whose goal was to provide baseline data to inform and guide the efforts of states, local districts, and schools during implementation of CCSSM. Over 12,000 teachers were surveyed, and results found that teachers appeared to be less aware of the CCSSM than the district curriculum coordinators surveyed. Although teachers conveyed some familiarity with CCSSM and were all from states that had adopted CCSSM, only 55% indicated that they were aware that CCSSM had been adopted in their state (Cogan et al., 2013). Like Amy, Beth, and Carly, frequent responses indicated that teachers used a combination of state, district standards, and/or NCTM standards, or some combination of these with CCSSM. Also consistent with Amy, Beth, and Carly were reports that primary grade teachers overall supported CCSSM but also voiced concerns about fitting everything from the textbook and differing standards into their instruction. Teachers were reluctant to omit anything for fear that their students would be lacking in the future (Cogan et al. 2013).

In order to address CCSS, all three participants stated that they were encouraged by their district to use the textbook as their curriculum, at least initially. Both districts adopted textbooks marketed to address the CCSSM. None of the participants were particularly pleased with their textbook series, but all three used it in varying degrees. Cogan et al. (2013) state that textbooks still exemplify the broad but shallow mathematics curriculum characterized in the U.S. in the Third International Mathematics and Science Study (TIMMS) (NCES, 1999; Schmidt, McKnight, and Raizen, 1997). All of the participants agreed that there were big gaps in what their textbook series presented and what students were expected to know. All participants added that time constraints were problematic, recalling having to create some of their own lessons in order to fill gaps left by their current textbook. These issues included searching for different resources, preparing materials for lessons, and time to complete a lesson without interruption. It will take some time for textbooks that are fully aligned with CCSSM to be widely available and to influence classroom practices. Until then, teachers will have the challenge of navigating differing visions of mathematics instruction that is reflected in textbooks, standards documents, and district interpretations of CCSSM (Cogan et al., 2013). This will be a particular challenge for teachers in the primary grades as a greater percentage of these teachers look to textbooks for guidance in their instruction (Cogan et al., 2013; Schmidt et al., 2012).

Professional Development

Although both districts had a plan in place for professional development in mathematics instruction, neither appears to have had a strong chain of communication, at least as perceived by the participating teachers. Meetings were rare and comprised of only one or two representatives from each school. Much of the communication was via e-mail, and participants were not familiar with their contact person. Much of the in-service components were related to the use of the new textbooks and sometimes an online option. Participants conveyed a lack of explanation and follow-up relating to professional development. Time for "training" is often limited to a planning period, so there is no time for practice or questions. All of the in-service trainings either related to the adoption of the new textbook or technology. No in-service trainings were scheduled relating to the use of manipulatives.

Unfortunately, professional development is typically considered after the fact (Hawley & Valli, 1999). Schools spend much less of their resources on professional development, but improving the instructional ability of the teachers is one of the most important characteristics of school improvement (Lampert & Ball, 1999). Professional development should focus on preparation for sustainable change in instructional practice as opposed to immediate, short-term change. While substantial content knowledge in mathematics is important, professional development should support lasting change and opportunities for reflection over a long period of time (Dewey, 1933; Senger, 1999). As noted in our discussions, often times teachers might dismiss approaches suggested in professional development because of their perceived constraints such as lack of Common Core requirements, limited time, inadequate text, or behavioral issues. Brief professional development sessions held during "planning periods," prior to school, or during teachers meetings are not adequate to elicit change. Research on effective professional development demonstrates the importance of supporting teacher learning, practice, and reflection (Hawley & Valli, 1999) and that successful professional development is the primary responsibility of administration. Successful initiatives are predominantly school-based and are an integral part of the day-to-day operations. Therefore, it is important that professional development should ensure that effective instructional practices are valued and consistent with the vision for mathematics instruction within the school setting.

In all three cases, participants felt that they were not part of an integral program that emphasized effective mathematical practices, providing them the ongoing support necessary to elicit change. Teachers reported that professional development in mathematics was either not present or did not include the entire population of mathematics instructors, but instead was held away from the school setting with only a select few. Further, those select few often communicated through e-mail, eliminating the one-to-one contact, opportunities for reflection, or ongoing support. Most teachers are in the early stages of their familiarity with CCSSM. Cogan et al., (2013) suggest that when teachers were asked what types of support would be most helpful in their efforts to address CCSSM in their classroom instruction they most often selected supports that involved providing teachers with practical assistance in developing ways to teach the new standards through some type of professional development or online, interactive website. The third most popular suggestion was online support for students. Only about 40 % of teachers surveyed selected textbooks.

The two newest teachers were in charge of the math planning for their team. Neither had the benefit of significant collaboration for math instruction. Only Carly felt that she had a strong collaborative team. She stated these collaborative efforts made a huge impact on how she teaches. There is evidence that suggest a positive relationship between teacher collaboration and students achievement. Guarino et al. (2006) found lower turnover rates among beginning teachers in schools with induction and mentoring programs that emphasized collegial support. Futernick (2007) suggested that teachers felt greater satisfaction when they were involved in decision-making and established strong collegial connections. Goddard et al. (2007) surveyed 452 elementary teachers to determine the extent to which they recounted working collectively to influence school improvement, curriculum and instruction, and professional development. They used mathematics and reading scores for 2,536 students to determine a relationship between teacher collaboration and student achievement and found a positive relationship between the two. Consequently, findings from this study are congruent with previous lines of research noting that collaborative planning communities have a direct impact on teacher satisfaction and student achievement.

How results of this study relate to teacher education and postsecondary involvement

How do we see reforms taking shape in relation to our current in-service teachers and the pre-service teachers preparing to go into the classroom? The Center on Education Policy (2013) conducted a study to report on states' strategies, policies and challenges during the implementation of CCSS. It focuses on state education agencies' (SEAs) partnerships with post-secondary education institutions regarding collaboration and partnerships around CCSS initiatives. In line with recommendations by Cochran-Smith (2003) that professional development programs should be integrated with coursework and an ongoing endeavor into teachers' first years in the classroom, key findings from the Center on Education Policy's study suggest that the majority of SEAs reported that they have forge partnerships with postsecondary education officials to assist in implementing CCSS. Further, a large majority of SEAs stated that working with higher education institutions in their state to transition to CCSS is a key challenge. The majority of SEA respondents are preparing to provide briefings on the CCSS for school of education faculty in colleges and universities, and that the majority of postsecondary institutions will review the CCSS in mathematics to determine if mastery of the standards indicates college readiness.

Recommendations for Future Research

Two broad ideas emerge from this study. First, Lesh et al.'s model of mathematical representations included manipulatives, pictures, real-life situations, written expression, and verbal expression. I found, through my observations of instruction and through discussions with the participants, that the findings from this study provided an opportunity to modify this model for future studies to include technology in order to reflect what I felt was observed in today's elementary classroom. Technology appeared in many forms during this study: moveable and/or manipulated pictures, written explanation, charts and graphs, displays of real-life situations, educational videos, and the use of verbal explanations of the use of symbolic representations. Since the teachers in this study used, and justified the use of, technology in ways that were different from other forms of representation, future research on representations should directly include technology as a distinct representation.

The second broad idea focuses on the power of collaborative professional development in eliciting thoughtful consideration of representation choices in mathematics instruction. Further research needs to investigate how districts are preparing beginning teachers to be effective agents of change in the classroom. How do beginning teachers collaborate with each other and with more experienced teachers within their school and district to address the needs that are specific to them? How do districts provide professional development that fosters the kind of collaboration needed to effect change in student achievement? How do leaders at the state and district levels provide assistance to all of their teachers, especially those new to the profession, to prepare them for the changes that are ushered in during transitions, such as adoption of new reforms?

Conclusions

Chapters IV and V represent the results and discussion from this research project. Chapter IV included a quantitative analysis of the three beginning teachers selected for in-depth study, addressing the first research question "How do beginning teachers make use of concrete representations for teaching mathematics?" and a qualitative analysis addressing the second research question, "In what ways do prior experiences and current teaching contexts impact beginning teachers use of concrete representations?" Results of the quantitative analysis revealed that although Carly made use of manipulatives more frequently in her classroom instruction than did her counterparts, none of the beginning teachers in the study used manipulatives for their primary representation during instruction. The qualitative analysis exposed that, although two of the participants felt that the notebook from their pre-service experience had the greatest impact on their current teaching practices, their comments and concerns during our discussions were not consistent with that statement.

The recurring themes in the discussions with participants as to some possible reasons for the participants' practices overall and particular differences between them were technology, behavioral issues, CCSSM, textbook-related matters, time constraints, professional development, and collegial collaboration. Because technology is ever emerging, I designed in Chapter IV an updated model of Lesh's (1987) model of representation to include technology as one form of representation. Concerns relating to the Common Core were intertwined with the adoption of textbooks that might address the CCSSM as well as professional development that might support teachers and students in the transition toward implementation of CCSSM. The most evident difference between the influences of Carly's instructional practices and her counterparts was the level of collaboration with her colleagues and the use of common time scheduling for instructional planning and professional development.

Based on my observations throughout this study, it seems that the current teaching contexts for beginning teachers had a greater influence on their teaching practices than their pre-service experiences. There appears to be a strong relationship between what is happening in beginning teacher's classrooms and teacher interactions within their teaching environment. This translates into a paradigm shift in how we might approach instruction for our pre-service teachers. The more we can immerse pre-service teachers into an authentic classroom setting during their learning process, the better the probability that they will transition into their own teaching environment embracing the effective strategies that are taught during their pre-service experience.

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APPENDIX A

Interview #1 Protocol

The researcher knows all of the participants and has worked with each participant before in various capacities during their time at Winthrop. Begin as you settle in at a table and get your materials out with conversation about how they are doing this year (i.e. how they feel about their class, the support they are getting from their co-workers and administration, getting up early, looking forward to the holiday break, etc.)

Researcher: Tell me about your experiences with mathematics in elementary school. What about middle school and high school?

Researcher: Do you remember which standardized tests you took (ACT, SAT, Praxis?) How did you do on the math portion of the tests you took?

Researcher: When you started Winthrop and had to take your math core courses, how did you feel about those? (*Possible Prompts if response is delayed:* Did you feel prepared to take these classes ? Did you feel like they were helpful?)

Researcher: How did you do in your mathematics methods class in college?

Researcher: Let's talk a little now about how you feel about your own understanding of specific mathematical concepts. What would be important for a teacher to know when teaching (*pick a few to discuss*):
Probability?

Fractions?

Decimals?

Algebraic concepts?

Geometry?

Measurement?

Elapsed time?

Place value?

Computation?

Researcher: Talk to me a little about your use of manipulatives…which manipulatives you use and how/when you use them.

Researcher: I am going to shift gears a little bit to talk about your experiences during your semester in math methods class sand your internship. Did your mathematics methods class instructor/professor use manipulatives frequently?

Researcher: Can you talk to me about some of the manipulatives you remember your instructor using and how he/she used them?

Researcher: What about your mentor during your internship? Did your mentor use manipulatives frequently?

Researcher: Can you talk to me about some of the manipulatives you remember your mentor using and how he/she used them?

Researcher: I noticed you mentioned ________________. Would you say that those experiences in ______________ (methods class, internship, current teaching environment, etc. depending on their previous response) influenced how you currently teach math? How so?

Researcher: That's about it. Thank you for taking the time to meet with me. I am going to leave the "Swivl" with you. Please record five consecutive math lessons and I will I will be in touch within the next two weeks to set up a time when I can come and observe you teaching a math lesson.

APPENDIX B

Interview #2 Protocol *(Code Name)*

Researcher: The last time we met, we talked a little about your views about your mathematics instructional practices and the practices of your math methods instructor and your internship mentor during your pre-service experience. Today we are going to focus on your current practices and what you find to be the things that are influencing how you taught mathematics this year.

Researcher: How was this first year for you?

Researcher: How did you feel about your math instruction this year?

(*Possible Prompts if response is delayed)*
Researcher: How did the pacing go?

Researcher: Your Teaching Strategies?

Researcher: How the students responded?

Researcher: Can you talk to me a little about the instructional decisions you made when planning your lessons? How did you decide to _______________?

Researcher: Are there other ways students could learn this topic?   (If yes) What might
that look like?

Researcher: What keeps you from doing that?

Researcher: Are there math manipulatives available that you could use for this?

Researcher: Have you used manipulatives and other representations very often in your instruction since the last time we talked?

Researcher: Why do you feel that is?

Researcher: How does the administration (school and district) support your mathematics instruction?

(*Possible Prompts if response is delayed))*
Researcher: In-service?

Researcher: Purchase of materials?

Researcher: Assessment?

Researcher: Familiarity with the Standards?

Researcher: I noticed that you mentioned ___________________________. Can you expand on that?

Researcher: You sound ____________________ *(adjective: pleased, proud, frustrated, etc.)* with that. *(Wait for further response.)*

Researcher: I think that about covers it. Is there anything that I didn't think about asking that you want to mention?

Researcher: Thank you for taking the time to meet with me. We will be meeting next Tuesday at 6:00 at Amici's for our Focus Group, and that should be the last thing we need to do.

APPENDIX C

Focus Group Questions Protocol

Opening question: Tell the group a little about your teaching situation (i.e. where you teach, grade level, number of months/years' experience you have had at this grade level and/or others)

Introductory Questions: Starting with kind of a round robin but then folks just kind of start jumping in, can you share with me some of the ways that you have used concrete representations (math manipulatives) in your classroom instruction this part year?

Transition Questions: You have mentioned ___________ (make eye contact with participant who mentioned each). Can you all talk to me a little about what people or events might have influenced those practices?

Key Questions:

1) What are some of the things that have been helpful in encouraging the use of manipulatives in your math instruction?
2) What do you find to be the biggest roadblocks in terms of using math manipulatives in your instruction?
3) What frustrations do you find in relation to your daily math instruction?

Ending Questions: *With 15 minutes remaining say*, “The overall purpose of this study is to look at 1) how beginning teachers make use of concrete representations and 2) in what ways do prior experiences and current teaching contexts impact beginning teachers’ use of concrete representations. Have we missed anything?”